L.A. ARTSIMOVICH

A PHYSICIST'S ABC ON PLASMA

Definition of Plasma ● Motion of Electrons and Ions in Plasma in the Absence of External Fields ● Plasma Behaviour in Electric Fields ● Plasma Behaviour in Magnetic Field ● The Effect of the Magnetic Field on Characteristics of the Plasma ● Results Derived from Magnetohydrodynamic Equations ● Development of Experimental Targets ● Conditions for Existence of a Plasma Ring ● The Theory of Stability ● Plasma Behaviour in Traps with Magnetic Mirrors ● Stability of Plasma Configurations ● Shear Stabilization ● Other Plasma Instabilities ●

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Л. А. Арцимович

ЧТО КАЖДЫЙ ФИЗИК ДОЛЖЕН ЗНАТЬ О ПЛАЗМЕ

Атомиздат Москва

L.A. ARTSIMOVICH

A PHYSICIST'S ABC ON PLASMA

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PREFACE

This small book written by the late Lev Artsimovich, a Full Member of the USSR Academy of Sciences, is the lecture course he delivered for the physicists interested in plasma physics. The book presents the fundamental information on the high-temperature plasma physics. These, at present largely well-established results, comprise, in effect, the sum of knowledge indispensable for any physicist with a wide enough sphere of interests.

However, it should be clearly understood that the structure of the high-temperature physics has just been outlined and is still being developed. A major contribution to this field of modern physics has been made by the Department of Plasma Physics, headed by L. Artsimovich, which is a part of the I. Kurchatov Institute of Atomic Energy. L. Artsimovich headed the research work with the tokamak-type installations, adiabatic traps, rapid pin-

ches and other machines designed for harnessing the high-temperature fusion reaction. He was directly involved in actual work in a number of these research projects and in development of the correct conceptual system and understanding of the plasma phenomena. This work, of course, could not fail to be reflected in the book; when discussing the experiments with tokamak-type installations and the adiabatic traps, L. Artsimovich described in more detail the most recent results available at that time. By now, some material in the book has become somewhat dated; for instance, plasmas generated in Tokamaks and adiabatic traps have now more impressive parameters and we have now a markedly better understanding of the processes in them. However, since the general conceptual system has not undergone any major changes we have deemed it unwise to alter the original author's presentation of ideas and, therefore, no significant changes have been made in the original text.

Hence, the material presented in the book comprises, in the opinion of L. Artsimovich, the fundamental results obtained through many years of experimental and theoretical research in the high-temperature plasma physics.

Prof. B. B. Kadomtsev

1.

DEFINITION OF PLASMA

The term *plasma* is applied to the ionized gas all (or a considerable part) of the atoms in which have lost one or several of their electrons and converted into positive ions. This definition of plasma as a specific state of matter is only a tentative one, it will be elaborated upon below. Generally, plasma can be assumed to be a mixture of three components — free electrons, positive ions and neutral atoms (or molecules).

Plasma is the most widespread state of matter in the universe. The Sun and the stars can be regarded as enormous lumps of hot plasma. The Earth's atmosphere is surrounded by a plasma envelope known as the ionosphere. The circumterrestrial space outside the ionosphere contains peculiar plasma formations — the so-called radiation belts. In the terrestrial environment we encounter plasma in laboratories and technology in various gas dischar-

ges since any gas discharge (lightning, spark, arc, etc.) invariably generates plasma.

The research in plasma physics was typically stimulated by potential practical applications of the results. At first, the scientists were interested in a plasma as a peculiar conductor of electric current and as a light source. Nowadays, we should consider the physical properties of plasma from another viewpoint and new aspects of plasma will be revealed to us. First, the plasma is the natural state of matter heated up to a very high temperature; and, secondly, the plasma is a dynamic system subjected to the action of electromagnetic forces. The new approaches in the studies of the plasma behaviour stem directly from the large-scale technological problems involving physics. The major problems are the controlled nuclear fusion and the magnetohydrodynamic conversion of heat energy into electric energy. In the near future plasma physics may play a significant role in accelerator technology.

The studies of plasma phenomena are of not only practical interest. The plasma is a material medium consisting of a group of particles which interact between themselves via the simplest mechanism, namely, the Coulomb electrostatic forces. A physicist has to understand the mechanisms of various processes which occur in a plasma proceeding from the known microstructure of plasma. The basic theoretical concepts here are extremely clear. Here we remain invariably within the domain of the classical physics since the quantum effects do not

play any noticeable part in the typical plasma. Nevertheless, the theoretical analysis of the plasma effects is far from being completed and many gaps still remain in our understanding of the plasma.

The major experimental efforts at present are aimed at developing the techniques for generating plasmas with high parameters, namely, high temperature and high density. Our subject of study — the high-temperature plasma — is produced by ourselves and we try to provide optimum conditions for the plasma existence when the plasma is in the quasineutral stable state.

Before continuing further, let us elaborate upon the definition of a plasma. The electric forces which bind the opposite charges in a plasma provide for its quasineutrality, that is, an approximate equality of the concentration of electrons and ions. Any separation of charges when electrons shift with respect to ions must give rise to electric fields which tend to compensate the perturbation. These fields are the higher, the higher is the particle concentration and for a high-density plasma they are very high, indeed.

To estimate a field produced by a perturbation of plasma neutrality, let us assume that charges have been completely separated in a certain volume so that it contains only one type of charges. The electric field in this volume satisfies the Poisson equation

$$\operatorname{div} \mathbf{E} = 4\pi\rho$$

where ρ is the charge density.

If the linear dimensions of the volume are of the

order of x and the concentration of the charged particles in the plasma is n, we obtain

$$\operatorname{div} \mathbf{E} \sim E/x \sim 4\pi ne$$

and, hence,

$$E \approx 4\pi nex$$

The variation of the plasma potential in the volume of charge separation is $V \approx Ex \approx 4\pi nex^2$.

Let us consider the following example. Assume that the fully ionized plasma is produced from hydrogen which is, initially, at the normal temperature and under the pressure of 1 mm Hg. A cubic centimetre of such a plasma contains $7 \cdot 10^{16}$ ions and electrons each. In these circumstances E is about 10^{11} V/cm. Therefore, if quasineutrality is sharply disturbed in a volume with a diameter of about 1 mm, the resulting electric field is more than 10^{10} V/cm and the potential difference across this volume is of the order of 10^9 V. Such a separation of charges is, clearly, unfeasible. Even in a plasma of a much lower density a sharp disturbance of quasineutrality in such volumes will be immediately suppressed by the developing electric fields. The electric field will push out the charges of one sign from the volume with separated charges and draw the opposite charges into it. However, in a small enough volume o plasma, quasineutrality may be violated if the electric field generated by an excess of particles of one sign is too weak to have a significant effect on the motion of particles. For a given concentration and temperature of the plasma, it is described by a characteristic linear parameter δ: when $x \ll \delta$,

then separation of charges within a volume with the linear size x does not produce a significant effect on the motion of particles; when $x \gg \delta$, then concentrations of the opposite charges in this volume are almost equal.

The characteristic length δ can be estimated in the following way. When the charges in the volume of the linear size δ are completely separated, the potential energy of a charged particle is of the order of the energy kT of the thermal (heat) motion of particles; here T is the plasma temperature in absolute degrees *. Thus, we obtain the relationship

$$U = \mathrm{eV} \sim 4\pi n e^2 \delta^2 \sim kT$$

Hence we find

$$\delta \approx [kT/(4\pi n e^2)]^{1/2} \tag{1}$$

The parameter δ is derived also from the analysis of electric field screening in a plasma. Assume that a "test" point charge q has been introduced into a plasma. The potential generated by this charge at a small enough distance r from it is q/r. However, polarization of plasma due to the electric field of the charge alters the form of the potential function at large distances from the charge.

* The thermal energy of plasma particles can also be expressed in electron-volts using the relationship $kT = e\theta/300$ eV $= 1.6 \cdot 10^{-12}\theta$ eV which yields $T = 11\,600\theta$ eV. The parameter θ is referred to as the plasma temperature expressed in electron-volts. The temperature defined in this way is identical to the parameter characterizing the thermal energy.

In a state of statistical equilibrium, the spatial distribution of electrons and ions in the vicinity of the test charge is given by the Boltzmann formula ($n \sim \exp(-U/kT)$, where U is the potential energy. U has opposite signs for ions and electrons). This can readily be seen to result in screening of the electric field in this region. The concentration of particles with the charges opposite to q is higher in the vicinity of the test charge q, that is, for comparatively high values of U/kT; this should result in a sharp drop in the electric field. Calculations using the Poisson equation and the Boltzmann distribution law show that at large distances from the charge q the potential decreases exponentially and the electric field is high only in the sphere with the radius of the order of δ around the charge q. Debye was the first to introduce the characteristic length δ in his study of strong electrolytes. Later, this concept was applied to plasma physics. The parameter δ is referred to as the Debye radius or the Debye length. Inserting the numerical values of the constants into Eq. (1) for δ, we obtain

$$\delta \approx 7\,(T/n)^{1/2} \tag{2}$$

Here T is the plasma temperature which is assumed, so far, to be the same for the electron and ion components of the plasma. While the Debye radius describes the spatial scale of the decompensation regions, the lifetime of these regions is found by dividing δ by the velocity of faster particles (electrons):

$$\tau = \delta/U_e = (kT/4\pi ne^2)^{1/2}(m_e/kT)^{1/2} = (m_e/4\pi ne^2)^{1/2}$$

The parameter $1/\tau$ has the dimensions of frequency and is identical to the frequency of the electrostatic plasma oscillations which occur in plasma when the electron groups are displaced from the equilibrium distribution. This frequency

$$\omega_0 = (4\pi ne^2/m_e)^{1/2} \tag{3}$$

is known as the plasma or Langmuir frequency.

The higher the density of plasma the smaller the sizes of decompensation regions and their lifetimes. In a high-density cold plasma quasineutrality can be violated only within a fairly small region. In a low-density hot plasma the Debye length can be considerably larger than the size of the plasma. If this is the case, the motions of ions and electrons are independent and no automatic equalization of concentrations of the opposite charges occurs.

We can now redefine the term plasma in the following way making use of the concept of the Debye radius — the plasma is an aggregation of freely moving particles with opposite charges, that is, an ionized gas is called the plasma if the Debye length is small compared to the size of the volume occupied by the gas. This definition was put forward by Langmuir who was the pioneer of the plasma science.

The two plasma parameters we have introduced — concentration and temperature — should be commented upon.

1. Generally, the electron and ion concentrations need not be identical since a plasma can contain not only singly charged but also multiply charged ions.

If the concentration of ions with a unit charge is denoted by n_1, the concentration of ions with the charge of 2 by n_2, and so on, the electron concentration n_e is given by $n_1 + 2n_2 + \dots$. However, below we shall mainly discuss the plasmas in which the electron and ion concentrations are identical. In particular, this is the case for the hydrogen plasma. Normally, it is not too complicated to take into account the effects of the multiply charged ions on the basic processes in the plasma.

2. The concept of the plasma temperature T is valid only if electrons and ions have the same kinetic energies. Generally, the plasma must be characterized by at least two temperatures — the electron temperature T_e and the ion temperature T_i. The plasmas which are generated in a laboratory environment or in technology have typically the temperature T_e which is usually much higher than T_i. The difference between T_e and T_i is due to an enormous difference between the masses of electrons and ions. The external electric power sources that generate a plasma (in various gas discharges) transfer the energy to electrons since it is the electrons that are the current carriers. Owing to collisions with rapidly moving electrons, the ions increase their thermal energy. In such collisions the fraction of the kinetic energy of electrons which can be transferred to the ion is no more than $4m_e/m_i$, where m_e and m_i are the masses of an electron and an ion. The mean fraction of the electron energy transferred to the ion in a collision is even lower. Since m_e is much less than m_i, the electron has to undergo very many

(thousands) collisions to transfer completely to the ions the excess energy it has. Since the thermal energy exchange between electrons and ions proceeds concurrently with the energy transfer from the electric power sources to electrons and at the same time the energy leaks from the plasma via various heat-transfer mechanisms, typical result is a sharp temperature difference between electrons and ions in an electric discharge in gases. This energy difference decreases, typically, with increasing plasma concentration since the frequency of collisions between electrons and ions in a given plasma volume increases with the squared concentration.

Under some special conditions T_i can become much higher than T_e in a strongly ionized plasma. For instance, such conditions exist in short-duration high-power pulse discharges which result in development and cumulation of shock waves in the plasma.

2.

MOTION OF ELECTRONS AND IONS IN PLASMA IN THE ABSENCE OF EXTERNAL FIELDS

The character of this motion is determined by the interaction between the particles. In the highly ionized plasma the principal type of interaction between the particles is the classical Rutherford scattering in the Coulomb field. We shall deal with three basic types of elementary scattering pro-

cesses, namely, scattering of electrons by ions, electrons by electrons and ions by ions. Other elementary processes either occur with photon emission and their probability is relatively low or they involve also the neutral particles and their significance diminishes with increasing degree of ionization. The processes of the first kind are illustrated by emission of bremsstrahlung electrons in the electron-ion collisions, the processes of the second kind are illustrated by the ionization phenomena and atom excitation by electron impact and by the charge exchange processes occurring between ions and atoms. If we deal with a non-hydrogen plasma we have, generally, to take into account the interaction between electrons and ions in various energy states. The intensity of radiation emitted by the excited ions can prove to be very high and can make a significant contribution to the energy balance of the plasma processes. We shall limit ourselves to analyzing particle interactions in a fully ionized plasma.

Let us assume that a charged "test" particle is travelling in the plasma (such a particle may be any plasma electron or ion whose path we shall trace). When travelling, this particle will be scattered in the Coulomb field of plasma electrons and ions which it encounters along its path. If we consider a light particle travelling among heavy ones (an electron among ions), then the scattering centres are assumed to be stationary. In this case the probability of scattering into a given angle is determined by the classical Rutherford formula.

Every scattering of a test particle travelling in the vicinity of the scattering centre results in the particle's trajectory being rotated by an angle θ, that is, its velocity along its initial direction of motion decreases from v to $v\cos\theta$. In most cases scattering occurs at large distances and typically results in a very small change in the trajectory (this is typical of the Rutherford scattering in the electric field of point charges!). Therefore, here we cannot visualize, as is customarily done in the kinetic theory of gases, the trajectory of a particle as being a broken line composed of straight segments or "paths" which connect the points of "collisions". Instead, we have a gently twisting line whose direction varies owing to the effect of the numerous but very weak impetuses due to "collisions" with other particles. In effect, these impetuses generate a continuous influence of the plasma "microfield" on a travelling particle. This microfield is a superposition of the electric fields of individual particles.

It is natural here to introduce the concept of the free path λ of a particle defining it as the distance at which the particle changes its initial direction of motion. This definition yields the following equation:

$$dv = -v dx/\lambda \tag{4}$$

Here dv is the mean variation of the velocity component along the initial direction of motion when travelling the distance dx. This definition can be used to express λ in terms of the integral over the angular distribution of scattered particles. If the

velocity vector rotates by the angle θ after collision, then the velocity component along the initial direction of motion decreases by $v(1 - \cos\theta)$. When a particle travels the distance dx in the volume with n scattering centres per 1 cm^3, the particle is scattered $nf(\theta)\,d\Omega\,dx$ times into the angle close to θ within the solid angle $d\Omega$. The function $f(\theta)$ is known as the effective differential cross-section of scattering and is determined by the nature of interaction forces. Hence, the mean value of dv is given by

$$dv = -vndx \int f(\theta)(1 - \cos\theta)\,d\Omega \qquad (5)$$

From Eqs. (4) and (5) we obtain

$$1/\lambda = n \int f(\theta)(1 - \cos\theta)\,d\Omega \qquad (6)$$

If the scattering centres are assumed to be stationary point charges, then the function $f(\theta)$ is given by the Rutherford formula

$$f(\theta) = 1/4\,(q_1q_2/m_1v^2)^2\,(1/\sin^4\theta/2) \qquad (7)$$

where q_1 is the charge of the test particle, m_1 is its mass and q_2 is the charge of the scattering centre.

Substitution of Eq. (7) into Eq. (6) and integration over angles from a minimum angle $\theta_{\min}$ to π yield

$$\lambda = 1/4\pi n\,(m_1v^2/q_1q_2)^2\,(1/\ln(2/\theta_{\min})) \qquad (8)$$

The value of $\theta_{\min}$ may be estimated as follows. The electric field of the scattering centre will be assu-

med to be a Coulomb one only at distances smaller than the Debye radius δ. At larger distances the potential decreases exponentially. Therefore, we have to ignore the collisions in which the particle passes the scattering centre at distances larger than δ. Now, note that the distances of approach of the order of δ correspond to extremely small scattering angles in the plasma. Under such conditions the trajectory of the test particle is almost a straight line and the closest approach distance is practically identical to the impact parameter. For small-angle scattering in the Coulomb field the impact parameter b is related to the angle θ by the well-known equation

$$\theta = q_1 q_2 / m v^2 b \tag{9}$$

Substitution of $b = \delta$ into Eq. (9) yields θ_{min}. Since this calculation yields a very high log quantity in Eq. (8) for interactions of particles in the plasma (in all the cases of interest it varies from 10^4 to 10^8), a rough approximation of the estimated minimum scattering angle practically does not affect the computation of λ.

The assumptions used for deriving the free path λ are valid when the test particle is an electron interacting with plasma ions. Denote by λ_{ei} the mean free path of electrons colliding with plasma ions. The path λ_{ei} is derived by averaging Eq. (8) over the energy spectrum of electrons. When all the plasma ions are singly charged and the energy distribution of electrons is given by the Maxwell formula, we obtain the following expression for the mean

free path:

$$\lambda_{ei} = 4.5 \cdot 10^5 \frac{T_e^2}{n} \frac{1}{L_c} \tag{10}$$

Here L_c is the so-called Coulomb logarithm which is derived by substituting into $\ln 2/\theta_{min}$ the minimum angle found from Eq. (9) for $b = \delta$, $mv^2 = 3kT_e$ and $q_2 = -q_1 = e$. The value of L_c varies from 10 to 20 for very wide variation ranges of n and T_e. Since fairly rough estimates of the parameters of particle collisions can be used in plasma physics we shall assume below that $L_c = 15$. Several other parameters describing the collisions between electrons and ions can also be introduced in addition to λ_{ei}. The effective cross-section σ_{ei} for such collisions is given by the relationship $\lambda_{ei} = 1/n\sigma_{ei}$, and the mean time period between two successive collisions is $\tau_{ei} = \lambda_{ei}/v_e$, where v_e is the mean thermal velocity of electrons. The collision frequency ν_{ei} is the reciprocal of τ_{ei}. The above parameters may be calculated from the following expressions (for $L_c = 15$):

$$\sigma_{ei} \approx 3 \cdot 10^{-5}/T_e^2; \ \tau_{ei} \approx 3 \cdot 10^{-2} T_e^{3/2}/n; \ \nu_{ei} = 20 n_e/T_e^{3/2} \tag{11}$$

The above parameters can be readily modified for describing the collisions with multiply charged ions. For these processes the effective cross-section σ_{ei} increases as the squared ion charge and the other parameters vary in a similar way.

The collisions between electrons and ions are the most significant particle interactions in the plasma;

in particular, they determine the mechanisms of such processes as electric conduction and diffusion.

To make a more comprehensive description of the Coulomb interaction between the plasma particles, we have to introduce also the parameters which will characterize the statistical effect of the collisions between identical particles (electron-electron and ion-ion collisions). Here the calculations are more complicated since the analysis of the elementary collisions should take into account the motion of scattering centres. However, this can affect only the numerical factor in the mean free path expression, while the temperature dependence should remain the same. For instance, the expression for λ_{ee} (the mean free path for the electron-electron collisions) must be the same as the expression for λ_{ei} differing only by a numerical factor fairly close to unity. The expression for λ_{ii} (the mean free path for the ion-ion collisions) is derived from the formula for λ_{ee} by substituting T_i for T_e. The parameters τ_{ee} and τ_{ei} are close to each other. The ratio τ_{ii}/τ_{ee} is $\sqrt{m_i/m_e}\ \sqrt{T_i^3/T_e^3}$. When the electron and ion temperatures are equal, the ion-ion collisions occur much rarer than the electron-electron or electron-ion collisions.

Let us summarize. The above analysis has attempted to describe the interactions of charged particles in the plasma in terms of the elementary kinetic theory of gases; we have replaced the smooth gradually weaving trajectories of electrons and ions by arbitrary broken lines and reduced the statistical effect of numerous weak collisions to one abstract

strong impact. The usefulness of such, not too rigorous approach, lies in the fact that the expressions for the mean free path, the mean time between two successive collisions, etc. help to visualize the basic physical processes in the plasma. There exists quite a rigorous technique for analyzing the Coulomb interactions between the plasma particles using the mathematics of the kinetic equations theory.

Let us now discuss the exchange of thermal energy between electrons and ions in the plasma. Let us, at first, consider the simplest case. Assume that a fast electron with the momentum $p = m_e v_e$ travels in the vicinity of a stationary ion and is scattered into the angle θ. In the process the ion receives the momentum $\Delta p = 2p \sin \theta/2$. As a result, the ion starts moving with the kinetic energy

$$\Delta W = (2p \sin \theta/2)^2/2m_i \tag{12}$$

To find the kinetic energy transferred by a fast electron to stationary ions per unit time, multiply ΔW by $nv_e f(\theta)d\Omega$ and integrate over angles. The result is

$$dW_e/dt = L_c(4\pi ne^4/m_i v_e) \tag{13}$$

(it is assumed here that the ions are singly charged). The expression for the transferred energy can be rewritten as

$$L_c(4\pi ne^4/m_i v_e) = (2m_e/m_i)(4\pi ne^4/m_e^2 v_e^3)\ (m_e v_e^2/2) =$$

$$= (2m_e/m_i)\nu_{ei} W_e \tag{14}$$

Here ν_{ei} is the number of collisions between the electrons with the kinetic energy W_e and stationary ions per unit time. The mean relative fraction of energy lost in one collision is $2m_e/m_i$ as should be expected from the model of the elastic collision between two spheres. The mean energy transferred from the plasma electron to the ions in 1 s is found from Eq. (13) by integrating over the Maxwell velocity distribution *:

$$-dW_e/dt = 1.2 \cdot 10^{-17} n/A \sqrt{T_e} \qquad (15)$$

where A is the atomic mass of the ionized gas. This equation is valid only for $T_e \gg T_i$; if T_e is comparable to T_i, Eq. (15) must be replaced by the following equation:

$$-dW_e/dt = (1.2 \cdot 10^{-17} n_e/A)(T_e - T_i)/T_e^{3/2} \qquad (16)$$

To estimate the efficiency of heat exchange between ions and electrons in the plasma, let us discuss a specific case. Let the electron temperature in the hydrogen plasma be maintained at 10^6 K and the electron concentration be $n = 10^{13}$. Under these conditions heating of the plasma from 0 K to $T_i =$ $= 1/2\ T_e$ takes 1.2 ms. Note that these calculations are meaningful only if in the time period under consideration the ions retain all the energy they have obtained. Since an ion with an energy corresponding to a few hundreds of thousand degrees travels about

* The Coulomb logarithm L_c is taken to be 15.

100 m in 1 ms, the heating of ions under such conditions necessitates efficient thermal insulation of the plasma.

A major characteristic of matter is the equation of state, that is, the relationship between the pressure, density and temperature. The equation of state for the plasma with isotropic distribution of velocities of charged particles has the same form as the equation of state for a two-component ideal gas:

$$p = n_e k (T_e + T_i) \tag{17}$$

where p is the plasma pressure which is the sum of electron and ion pressures. The equation of state is known to have such a simple form only if the mutual potential energy of particles is negligibly small compared to their thermal energy. In the plasma, this is practically always the case. Indeed, the potential energy of a charged particle in the fluctuating plasma microfield should be of the order of e^2/a, where a is the mean distance from a given particle to the next one ($a \sim 1/\sqrt[3]{2n_e}$). It can be readily seen that the ratio

$$(e^2/a)/(kT) \sim \sqrt[3]{n_e}\, e^2/(kT)$$

is much smaller than unity except when we consider a plasma with the particle concentration of 10^{18} and higher and the temperature not lower than 10^4 °K.

The energy distribution of the gas particles is usually assumed to have the Maxwellian form. This assumption cannot be automatically applied

to the plasma. The Maxwellian energy spectrum is established owing to collisions between the gas particles.

The Maxwell distribution in a wide energy range can be established for a given group of particles with an arbitrary initial energy distribution, that is, the Maxwellian "tail" for the particles with energies much higher than kT can grow only during a certain time period in which the particles must undergo, in the average, several collisions with each other and these collisions must be between the identical particles. For instance, the energy distribution of electrons will become practically Maxwellian after a time period which is about 10 times the mean time τ_{ee} between two successive electron-electron collisions. For ions the Maxwellian distribution is established after about ten ion-ion collisions Thus, when T_i is about T_e "maxwellization" for ions proceeds much slower than for electrons. However, for each plasma component, maxwellization due to collisions between the identical particles proceeds faster than the development of thermal equilibrium between both components. Therefore, both ions and electrons in the plasma have the Maxwell distributions but with two different temperatures. At the same time, if the charged plasma particles have a short lifetime, the energy distribution of ions in the range of energies much higher than kT can differ sharply from the Maxwellian distribution (the number of such fast ions will be much less than it should be according to the Maxwell formula).

3.

PLASMA BEHAVIOUR IN ELECTRIC FIELDS

An electric field or a pressure gradient in a plasma gives rise to directed particle fluxes (flows) — an electric current in the first case and a diffusion flux in the second case. When electric current is flowing in plasma, the ions can be assumed stationary. The current is carried by electrons. In the simplest case of constant current there should be established an equilibrium between the force of the electric field acting on electrons and the decelerating force due to collisions between electrons and ions. The latter force is given by the mean directed momentum lost by electrons in collisions with ions per unit time. An electron has ν_{ei} collisions in 1 s and in each collision it loses the momentum $m_e u$, where u is the directed electron velocity. Hence, the decelerating force is $m_e u \nu_{ei}$ and the equilibrium is established when

$$eE = mu\nu_{ei} \tag{18}$$

The current density in plasma is

$$j = neu \tag{19}$$

Hence we obtain

$$j = ne^2 E/m_e \nu_{ei} = ne^2 \tau_{ei} E/m_e \tag{20}$$

Eq. (20) is the Ohm law for a plasma. The parameter

$$\sigma_E = ne^2 \tau_{ei}/m_e \tag{21}$$

is the plasma conductivity. Insertion of the expression for τ_{ei} and the numerical values of constants yields for $L_c = 15$

$$\sigma_E \approx 10^7 T_e^{3/2} \quad \text{CGSE units} \tag{22}$$

This formula is valid for a fully ionized plasma with singly charged ions (the hydrogen plasma). Note that conductivity grows rapidly with increasing T_e. At temperatures about 10^8 K, the conductivity of the hydrogen plasma must be higher than the conductivity of copper at room temperature more than by an order of magnitude. If plasma contains multiply charged ions its conductivity is considerably lower. In the general case when the plasma contains ions with the charges $z_1, z_2, z_3, \ldots, z_n$ with the concentrations $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$, Eq. (22) for the conductivity is transformed into

$$\sigma_E \approx 1 \cdot 10^7 \frac{\sum_k \alpha_k z_k}{\sum_k \alpha_k z_k^2} T_e^{3/2} \tag{23}$$

Let us now discuss briefly the application of the Ohm's law to a plasma. This law is valid if in plasma there is an equilibrium between the forces of the electric field acting on electrons and the decelerating forces. However, can such an equilibrium be established under any conditions?

The decelerating force action on an electron accelerated by the field is the lower the higher is the electron's velocity. Take an electron from the far

tail of the Maxwell distribution ($W_e \gg kT_e$). The directed velocity component acquired by the electron between two "collisions" with ions is proportional to τ_{ei} and, therefore, increases as v^3. Therefore, if the thermal velocity v of our electron is high enough, its directed velocity u can become as high as v or even higher. Under these conditions it is no longer possible to use a simplified model in which the electron accelerates to a relativity low directed velocity along its free path and then completely loses it in an instantaneous strong "impact". Actually, acceleration and deceleration of the electron are concurrent. While the electron is acquiring the directed velocity, the Rutherford scattering on ions is gradually changing the direction of its motion. The electric field tends to straighten the electron's trajectory while the interaction with ions is bending it. If the increase in the directed velocity component is not compensated by scattering, the equilibrium of forces cannot be established and the electron should accelerate continuously so that its energy continuously increases. With increasing electron energy, the decelerating force decreases, hence, an electron involved in continuous acceleration by the field will be accelerated as long as it remains in the field. The field accelerates continuously those plasma electrons which have acquired along the free path λ an additional velocity u which is higher than their initial velocity v. This condition for continuous acceleration may be written as follows:

$$eE\tau_{ei}/m_e > v \qquad (24)$$

Since τ_{ei} is proportional to v^3/n, Eq. (24) shows that continuous acceleration occurs only when EW_e/n is higher than a certain boundary value. As can easily be seen, for the hydrogen plasma this boundary value is of the order of $3\cdot10^{-12}$, if E is measured in V/cm and W_e in eV. In the plasma experiments the condition (24) is typically satisfied only for electrons with the energy much higher than kT_e. The share of such electrons in the electron component is very low. In this case, the current comprising the overwhelming majority of plasma electrons satisfies the Ohm law. However, there will exist in the plasma the current of the accelerated electrons for which the Ohm law is not valid. For large values of E/n, the condition (24) will be satisfied also for the electrons with the medium thermal velocity. In this case, continuous acceleration can involve the major part of the plasma electrons and the Ohm law will be sharply violated. As calculations show, the continuous acceleration process develops markedly when the ratio of the mean value of u for the plasma as a whole to the mean thermal velocity of electrons is greater than 0.1. The ratio u/v increases with v^2 so that when u/v for the electrons with the mean thermal velocity is 0.1, for the electrons with the energy of 10 kT_e the velocity u is close to v. Such electrons are close to the threshold of continuous acceleration. It may be assumed that continuous acceleration of electrons is observed in ring-type electric discharges when a plasma is generated inside a toroidal chamber and accelerated by a cyclic electric field. In these

experiments under special conditions a certain (relatively small) group of plasma electrons has been observed to be accelerated to very high energies at comparatively low voltages across the plasma ring.

A more detailed analysis of the accelerated electron flows indicates that these flows can generate and amplify a variety of oscillations and waves in the plasma and transfer their energy to them. This gives rise to a new mechanism of braking the accelerated praticles which puts a stop to acceleration after the plasma electrons have acquired a certain additional energy of a directed motion. This automatic braking mechanism makes it impossible for all the plasma electrons to be continuously accelerated. However, Eq. (22) cannot be used in this case for determining the plasma conductivity since braking of electrons through interaction with the waves must increase the resistance. The measured plasma conductivities in the ring systems agree with the above considerations. For high plasma density and relatively low electric field the experimental values of σ_E coincide to the measurement accuracy with the values found from Eq. (22). The resistances are found to be abnormally high in low-density plasma at high temperatures.

The peculiarities of the plasma are especially noticeable when we consider its behaviour under the effect of high-frequency electric fields. In this case the mechanical inertia gains particular significance. Let us consider the simplest case. Let the plasma be under the effect of the electric field

$E = E_0 \exp(i\omega t)$ (complex notation simplifies the analysis). If the field frequency is high enough that during one period of field variation the probability of collision between an electron and ions is small enough, we can neglect the braking force in the first approximation in our analysis of the motion of electrons. Hence the equation of motion can be written as

$$m_e \ddot{x} = -eE_0 \exp(i\omega t) \tag{25}$$

where x is the coordinate along the field. Integration of Eq. (25) yields the velocity

$$u = -\dot{x} = eE_0 \exp(i\omega t)/(im_e\omega) \tag{26}$$

Hence we obtain

$$j = neu = \frac{1}{i}\frac{ne^2}{m_e\omega} E \tag{27}$$

Eq. (27) can be rewritten as

$$E = im_e\omega j/(ne^2) \tag{28}$$

Thus, the phase of a high-frequency field is shifted by 90° ahead with respect to the plasma current. This means that plasma in a high-frequency field possesses its own "non-magnetic" inductance which is due to the inertia of electrons. For low concentrations the non-magnetic inductance of a plasma conductor can be higher than its normal ("magnetic") inductance. Let us analyze the conditions required for that. For a cylindrical uniform plasma conductor with the radius a, the non-magnetic inductance per unit

length is given by (in the CGSE units)

$$\frac{1}{\omega}\left|\frac{E}{I}\right| = \frac{1}{\pi a^2\omega}\left|\frac{E}{J}\right| = \frac{m_e}{\pi a^2 n e^2} = \frac{m_e}{Ne^2}$$

where N is the number of electrons per 1 cm of plasma length. The normal inductance (self-inductance) for a conductor 1 cm long is of the order of unity in the CGSM units. In the CGSE units the inductance is of the order of $1/c^2$. Hence, the ratio between the normal inductance and the non-magnetic inductance for plasma is

$$Ne^2/m_ec^2 \approx 3\cdot 10^{-13}N$$

This ratio has a very simple physical meaning: it is the full number of electrons contained in a plasma conductor segment with the length of the classical electron radius ($r_e = e^2/m_ec^2$).

Eq. (27) relating the current density to the high-frequency field can be extended to take into account the braking of electrons due to collisions. Thus, the following relationship is obtained between E and j:

$$E = j\,(\rho + i\omega L_e) \qquad (29)$$

Here $\rho = 1/\sigma_E$ and $L_e = m_e/(ne^2)$ is the non-magnetic inductance.

Some comments should be made here on the dielectric properties of plasma. These properties are also related to the motion of electrons under the effect of external electric field. In the above simple case of the external electric field $E = E_0 \exp(i\omega t)$, the acceleration $\ddot{x}$ and the velocity $\dot{x}$ of an electron are given by Eqs. (25) and (26). Further integration

over time yields the displacement

$$x = eE_0 \exp(i\omega t)/(m_e\omega^2) \tag{30}$$

Eq. (30) shows that the electron displacement x has a phase shift of 180° with respect to the force $-eE$ acting upon the electron. This effect is the reverse of that encountered in normal solid dielectrics. When the phase shift between the displacement of the charge and the force acting upon it is 180°, the polarization of the material is opposite to the field and, hence, the dielectric constant is less than unity. The dielectric constant can be expressed in terms of the electric moment p of the unit volume in a well-known way:

$$\varepsilon = 1 + 4\pi p/E \tag{31}$$

We have in the plasma

$$p = -nex \tag{32}$$

where x is the displacement of electrons (the displacement of ions can be neglected since it is very small). Eqs. (30)-(32) yield

$$\varepsilon = 1 - \omega_0^2/\omega^2 \tag{33}$$

Here ω_0 is the plasma frequency given by Eq. (3). When ω is less than ω_0, the dielectric constant is negative. A consequence of this is that the electromagnetic waves with the frequencies lower than ω_0 cannot penetrate the plasma and are totally reflected from its surface.

The diffusion processes in a plasma are, to a certain extent, similar to those in normal gas. A diffu-

sion coefficient can be determined for each plasma component; similar to the kinetic gas theory the diffusion coefficient is $\lambda v_t/3$, where λ is the mean free path and v_t is the mean thermal velocity of the species.

However, the plasma quasineutrality condition necessitates that the electron flux be equal to the ion flux across any surface in the plasma. Since electrons have a much higher diffusion coefficient than ions (the ratio is the same as the ratio between the velocities for equal free paths), an electron concentration gradient at first will give rise to an electron flux from a higher-concentration region to a lower-concentration region greater than the ion flux. This will result in polarization of the plasma and in development of an electric field that will brake the electrons and accelerate the ions. Simple calculations show that the electron flux will be greatly decreased and the ion flux will be somewhat increased. This effect is known as the ambipolar diffusion. It is frequently found in the experiments with relatively weakly ionized cold plasma with diffusion of electrons and ions through neutral gas.

The heat conductivity of plasma is determined by energy transfer in collisions between particles with different thermal energies in a region with temperature gradient. The decisive part in heat transfer should be played by the collisions between the electrons since their collision frequency is high. The electron heat conductivity coefficient in a fully ionized plasma with singly charged ions can be cal-

culated from the following formula:

$$\delta_h \approx 1.2 \cdot 10^{-6} T^{5/2} \tag{34}$$

where δ_h is expressed in erg/cm·s·deg. Eq. (34) is valid when the electron mean free path is small as compared with the size of the region where the temperature gradient is established. Note that the relationship between the heat conductivity coefficient δ_h and the electric conductivity coefficient σ_E is given by the well-known Wiedemann-Franz law. The heat conductivity of a fully ionized plasma increases very rapidly with temperature. Even when T is about 10^5 K the heat conductivity of the hydrogen plasma is higher than the heat conductivity of silver at room temperature.

4.

PLASMA BEHAVIOUR IN MAGNETIC FIELD

A physicist making his first acquaintance with the basic plasma concepts can readily feel some disappointment at the beginning since everything seems to be too clear and almost self-evident. The work in plasma physics seems to be lacking in excitement, it looks like doing exercises with already known solutions. However, this attitude cannot develop when one considers the behaviour of a plasma in a magnetic field.

Due to the effect of the magnetic field the plasma becomes non-isotropic and radically changes all its

properties. A magnetic field can be used to establish closed plasma configurations contained in a limited volume and, so to say, suspended in a vacuum. This is something that is quite untypical of gas. Under such conditions the plasma properties are closer to those of liquids. At the same time such a plasma possesses the properties which distinguish it from all other states of matter. Ultimately, all the characteristics of plasma processes depend on the motion of particles; therefore, before we attempt the macroscopic description of magnetic properties of plasma, we have to consider in brief the effect of the magnetic field on the motion of electrons and ions.

A charged particle in a uniform magnetic field is generally known to travel along a helical line. The projection of the trajectory on the plane perpendicular to the magnetic induction vector **B** is a circle with the radius $\rho = mv_\perp c/(qB)$, where $v_\perp$ is the transverse component of the particle's velocity. This is the so-called Larmor circle. The motion along this circle is the rotation with the Larmor frequency $\omega_B = qB/(mc)$. The particle also travels along the lines of force with the constant velocity $v_\parallel$.

Generally, the magnetic field is not uniform. However, in non-uniform fields typically considered in plasma physics, induction **B** has almost a constant value and sense at the distances of the order of the Larmor radius of particles, that is, microscopic variations of the magnetic field are very small. Let us find how such weak non-uniformity of the field affects the motion of particles.

At first, assume that the field varies along the field line. The trajectory of the particle travelling along this line noticeably changes its shape within a segment in which the magnetic induction B increases or decreases significantly. When the particle travels towards the increasing field, the trajectory becomes more steep and it can be compared to a spring being compressed. When a particle travels towards a decreasing field, its trajectory becomes less steep.

This effect can be readily explained. A charged particle which rotates along the Larmor circle gives rise to a circular current so that it is equivalent to an elementary diamagnetic with the magnetic moment $\mu = W_\perp/B$, where $W_\perp$ is the kinetic energy of the transverse motion. Indeed, the Ampere theory yields

$$\mu = \frac{i}{c}\pi\rho^2 = \frac{q\omega}{2c}\rho^2 = \frac{q}{2c}\frac{qBm^2v_\perp^2c^2}{mcq^2B^2} = \frac{mv_\perp^2}{2B} = \frac{W_\perp}{B} \qquad (35)$$

The magnetic field whose intensity varies along the lines of force acts upon a diamagnetic with the force

$$F = -\mu dB/dl \qquad (36)$$

(here differentiation is done in the direction of the field). The action of this force results in variation of the longitudinal velocity $v_\parallel$ according to the relationship:

$$m\frac{dv_\parallel}{dt} = -\frac{W_\perp}{B}\frac{dB}{dl} \qquad (37)$$

Multiplication of both sides of Eq. (37) by $v_{\parallel}$ yields

$$\frac{dW_{\parallel}}{dt}=-\frac{W_{\perp}}{B}\frac{dB}{dl}\frac{dl}{dt}=-\frac{W_{\perp}}{B}\frac{dB}{dt} \tag{38}$$

When a particle travels in a magnetic field, $W_{\perp}+$ $+W_{\parallel}$ is constant. Therefore, Eq. (38) can be transformed into

$$\frac{dW_{\perp}}{dt}=\frac{W_{\perp}}{B}\frac{dB}{dt} \tag{39}$$

Hence, we find

$$dW_{\perp}/W = dB/B;\ \ W_{\perp}/B = \text{const} \tag{40}$$

Thus, the ratio $W_{\perp}/B$ is constant when a charged particle travels in the magnetic field whose intensity variation along the field lines is not too sharp.

The constant $W_{\perp}/B$ is usually referred to as the adiabatic invariant of motion. This emphasizes the fact that the particle travels in a slowly changing magnetic field. The kinetic energy of the transverse motion is $W_{\perp} = W_0 \sin^2\alpha$, where W_0 is the full energy of the particle and α is the angle between the velocity direction and the field line. Since W_0 is constant and $W_{\perp}/B$ is the adiabatic invariant, the ratio $\sin^2\alpha/B$ is also an adiabatic invariant. This shows that the slope increases with increasing B and in the region with a stronger magnetic field the helical trajectory becomes more steep as discussed above.

Assume that at a certain trajectory point $\alpha = \alpha_0$ and $B = B_0$. Using these initial conditions, we can find the value of α at any trajectory point from

the relationship $\sin^2\alpha/B = \sin^2\alpha_0/B_0$, that is,

$$\sin\alpha = \sin\alpha_0 \sqrt{B/B_0} \qquad (41)$$

When a particle travelling towards the increasing field comes to the point where $B = B_0/\sin^2\alpha_0$, then the angle α becomes 90° and hence the longitudinal velocity $v_{||}$ vanishes. This means that at this point the direction of the longitudinal motion is reversed. The particle is reflected from the region of high field towards the region of a lower field.

Thus, high-field regions under certain conditions can act as some magnetic mirrors for charged particles. For instance, if the field increases in opposite directions from a certain intermediate region, a charged particle can be blocked between two magnetic mirrors; it will oscillate along the field lines within a restricted space region. The particles with a large enough angle α ($\sin\alpha > \sqrt{B_{min}/B_{max}}$) will be confined in this region.

Let us now consider the motion of particles in a non-uniform field whose intensity varies perpendicular to the field lines. Let us at first consider the simplest case when the velocity is perpendicular to the magnetic field. Fig. 1 illustrates the particle's trajectory under such circumstances. The magnetic field is perpendicular to the plane of Fig. 1.

The field increases along the axis x. Here the particle's trajectory projection in the xy plane is not a circle, since the Larmor radius at the right is less than the radius at the left. Clearly, the trajectory is not closed after a complete cycle of rotation. Each

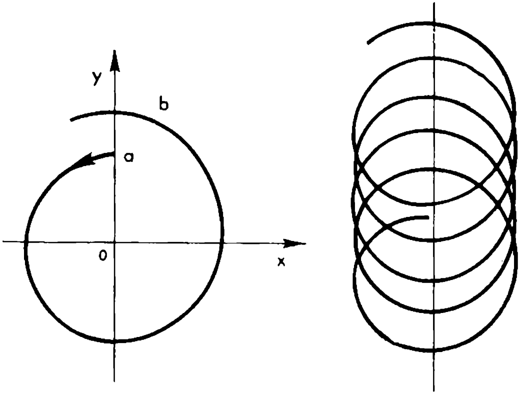

Fig. 1.

Drift of a charged particle in a non-uniform transverse magnetic field.

rotation cycle of the particle comprises a loop and with each loop the particle spirals a distance Δy along the axis y, that is, perpendicular to the magnetic field gradient. The particles of opposite charges spiral in opposite directions along the axis y. After several rotation cycles, it can readily be seen that the particle's trajectory is a path of loops stretching parallel to the axis y. Such a motion is known as the magnetic drift. The drift velocity of the particle is small compared to its Larmor velocity (it is assumed here that variation of the field intensity at the distances of the order of the Larmor radius is small).

Let us note the following feature of the drift motion. In the course of this motion the particle does not travel towards stronger or weaker fields. On the contrary, the particle travels along a narrow path which stretches within a constant-field region. Hence, the parameter $W_{\perp}/B$ is an adiabatic invariant in the case of the magnetic drift.

The magnetic drift velocity v_d is $\Delta y/T$, where T is the Larmor rotation period. The ratio between v_d and the rotation velocity, clearly, must be of the order of ρ/l, where ρ is the Larmor radius and l is the characteristic dimension of non-uniformity of the magnetic field:

$$1/l \sim (1/B)\,(dB/dx) \tag{42}$$

Hence we find

$$v_d \sim v_{\perp} \frac{\rho}{B} \frac{dB}{dx} \sim \frac{mv_{\perp}^2 c}{qB^2} \frac{dB}{dx} \tag{43}$$

Calculations which we omit here confirm Eq. (42) and yield the following expression for v_d:

$$v_d = \frac{1}{2} \frac{mv_{\perp}^2 c}{qB^2} \frac{dB}{dx} \tag{44}$$

Eq. (44) gives the velocity of the motion in the plane perpendicular to the induction **B**. However, the drift motion in the non-uniform magnetic field can also be related to the longitudinal velocity $v_{\parallel}$ of the particle. The mechanism of such a drift is illustrated by Fig. 2. The thick lines in Fig. 2 represent the field lines of the non-uniform magnetic field which are generally curved. The Lorentz force

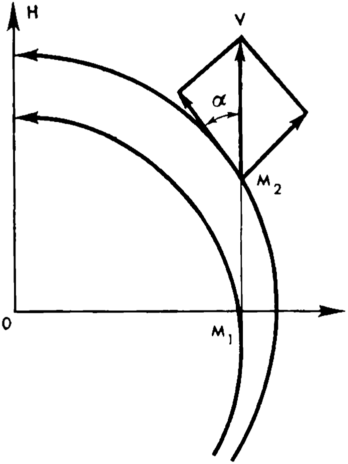

Fig. 2.

Drift of a charged particle in a non-uniform magnetic field with the velocity parallel to the field.

is zero at the point M_1 where the particle's velocity is parallel to **B**. However, the particle will continue inertial motion and slip off the field line; this will give rise to a small velocity component perpendicular to the field at the point M_2. The transverse velocity will automatically give rise to the Lorentz force. This force gives rise to a drift velocity which is perpendicular to the plane of Fig. 2. Consideration of special cases (for instance, the motion of particles in a magnetic field generated by a recti-

linear current) readily shows that the particle's drift due to the longitudinal velocity $v_{\parallel}$ has the same direction as the drift due to the transverse velocity $v_{\perp}$.

The following expression can be derived for the drift velocity in the general case, when $v_{\parallel} \neq 0$ and $v_{\perp} \neq 0$:

$$v_d = \frac{1}{\omega_B R} (v_{\parallel}^2 + v_{\perp}^2/2) \tag{45}$$

Here, ω_B is the Larmor frequency and R is the radius of curvature of the field line. The velocity vector has the sense of the vector product [**B** $\times$ **R**]. Eq. (45) is valid when the following two conditions are met: $v_d \ll \sqrt{v_{\perp}^2 + v_{\parallel}^2}$, that is, the drift velocity is much smaller than the velocity of motion of the particle; and the current density in the region where the particle travels is zero (or low enough). When these conditions are satisfied, the relationship between the gradient of the field intensity and the radius of curvature of the field line is simple enough to make possible derivation of Eq. (45). If the second condition is not satisfied, the expression for the drift velocity has a more complicated form. Note that both components of the particle's velocity remain constant in the course of the drift motion. Since the particle always drifts perpendicular to the gradient of B, the value of B along the drift path remains constant. Therefore the drift motion does not violate the invariance of the parameter $W_{\perp}/B$.

From the above it follows that the motion of a charged particle in the non-uniform magnetic

field can generally be expressed as a superposition of three types of motion:

(i) rotation along the Larmor circle with the velocity $v_{\perp}$;

(ii) motion of the centre of the Larmor circle along the field line with the velocity $v_{\parallel}$;

(iii) drift motion of the centre of the Larmor circle in the direction perpendicular to **B** and grad **B**.

The instantaneous positions of the centre of the Larmor circle comprise the axial line of the trajectory which, at the same time, can be considered to be an averaged path of the particle. The form of this line is a fundamental geometric characteristic of the motion of a particle in the magnetic field.

The nature itself provides us with an excellent illustration of the motion of charged particles in non-uniform magnetic fields, namely, the so-called earth radiation belts. The radiation belts were found by the first earth satellites and space rockets; they consist of high-energy electrons and ions trapped in the earth's magnetosphere. The motion of electrons and ions of the radiation belts in the magnetic field of the earth is rather complicated. If we ignore the Larmor rotation and consider only the averaged trajectories, the resulting paths are shown by the thick line in Fig. 3. Charged particles in the magnetic field of the earth oscillate along the field lines and are reflected from higher-field regions near the magnetic poles. A slower drift due to the curvature of the field lines is superimposed on this oscillatory motion. The drift motion is in the azimuthal direction. The particles of opposite charges

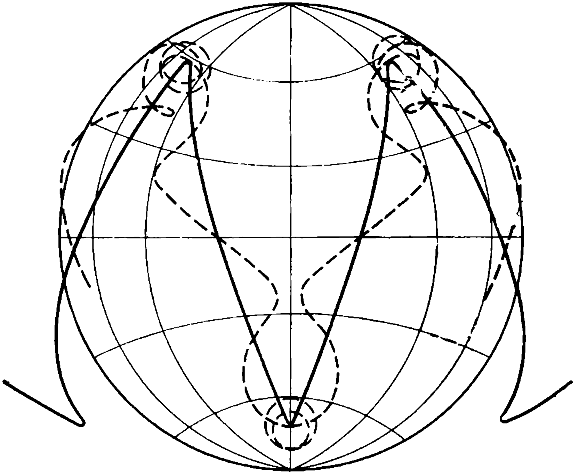

Fig. 3.

Motion of charged particles in the magnetic field of the earth. The thick line is the averaged trajectory.

travel in opposite directions going around the globe from east to west and from west to east.

To end this brief summary of motion of particles in the magnetic fields, we should consider the situation when, along with the magnetic force, there is a force of some other origin acting on the travelling particle. Of the highest interest is the case when such a non-magnetic force is perpendicular to **B**. We shall omit here very simple calculations which

show that a force **F** perpendicular to **B** gives rise to the drift with the velocity

$$v_d = \frac{c}{qB^2}[(\mathbf{F} \times \mathbf{B})] \qquad (46)$$

When the force is due to an electric field **E**, we have $\mathbf{F} = q\mathbf{E}$ and

$$v_d = \frac{c}{B^2}[\mathbf{E} \times \mathbf{B}] \qquad (47)$$

Here the drift velocity does not depend on the charge and sign of the particle. The drift motion of this type also does not violate the adiabatic invariance of $W_\perp/B$ (the particle travels perpendicular to the force **F** and therefore its mean kinetic energy during one Larmor cycle remains constant).

5.

THE EFFECT OF THE MAGNETIC FIELD ON CHARACTERISTICS OF THE PLASMA

Let us now analyze the effect of the magnetic field on the properties of plasma. Let us assume that a moderate-density plasma is in a strong magnetic field. Between two Coulomb collisions, each charged particle of the plasma travels in a helical trajectory along the field. If the field is uniform, the axial line of the trajectory is a field line. The particles can travel perpendicular to the field only owing to the Coulomb collisions. Each collision makes a particle

travel a distance of the order of the Larmor radius. If the collisions are rare (low-density high-temperature plasma) the particles are, so to say, tied to the field lines. Such a plasma is known as a "magnetized" plasma. The degree of "magnetization" is given by the ratio λ/ρ, where λ is the mean free path and ρ is the mean Larmor radius. When $\lambda/\rho \gg 1$ (rare collisions and high field), the particle can travel an appreciable distance perpendicular to the field only if it travels a considerable distance along the field. When $\lambda/\rho \ll 1$ (high-density plasma, weak field) the motion of particles is practically isotropic; this means that the magnetic field has a weak effect on the behaviour of the plasma.

The degree of magnetization can also be expressed in the following form:

$$\lambda/\rho \sim \frac{\lambda}{v} \frac{qB}{mc} = \omega_B \tau$$

where ω_B is the Larmor frequency and τ is the mean time between two successive collisions. The electron and ion components of the plasma have different magnetizations. Under normal conditions the value of $\omega_B\tau$ for the plasma electrons is much higher than for the plasma ions (when $T_i \sim T_e$, the ratio between these parameters is of the order of $\sqrt{m_i/m_e}$). Hence, magnetization of electrons is higher than magnetization of ions. It can happen, for instance, that electrons are magnetized and therefore travel freely along the field lines, while the magnetic field by itself does not have a noticeable effect on the motion of ions. Under such circumstances,

electrons are tied to the magnetic field, while ions are confined to the same space region by the electric field of the electron plasma component.

But we are primarily interested in high-temperature fully ionized plasma; in such a plasma the typical situation is when both electron and ion components are magnetized. The main parameters of experimental hot plasmas vary in the following ranges: concentration — from 10^{10} to 10^{14}, electron and ion temperatures — from 10^6 to 10^7 degrees, and magnetic field intensity — tens of kilogausses. For these ranges of n, T and B the magnetization parameter $\omega_B\tau$ for the electrons varies from 10^5 to 10^{11} and for the hydrogen ions, from 10^3 to 10^9.

Since a strong magnetic field restricts the motion of plasma particles in the plane perpendicular to the vector $\mathbf{B}$, such a field can be used as a screen preventing the contact between the plasma and the walls of the vessel containing the plasma (Fig. 4). A cylindrical column of fully ionized plasma is inside a chamber with a strong magnetic field. The space between the plasma surface and the chamber walls contains only vacuum and the magnetic field lines. The electrons and ions of the plasma cannot penetrate this space. Thus, a strong magnetic field provides for an effective thermal insulation of plasma. But strictly speaking, this is not an equilibrium state. Owing to Coulomb collisions, the plasma will, sooner or later, spread all over the chamber's volume up to its walls. The lifetime of an insulated plasma column is determined by the diffusion rate of the plasma particles in the magnetic

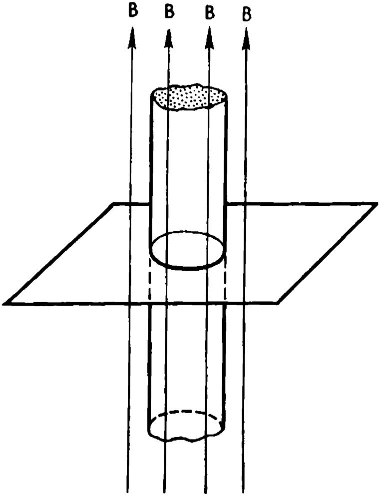

Fig. 4.

A plasma column in a magnetic field.

field (in the plane perpendicular to **B**). The diffusion theory shows that this lifetime is of the order of $a^2/\sigma D_\perp$, where a is the radius of the plasma column and $D_\perp$ is the transverse diffusion coefficient. A very rough estimate of $D_\perp$ can result from the following simple calculations. During the period τ of one free path, the particle in the average makes one Coulomb collision which results in the particle's displacement in the plane perpendicular to **B** to a distance of the order of the Larmor radius ρ. To

find the resulting displacement after numerous collisions, we have to add up the squares of the individual displacements according to statistical relationships. Thus, the displacement of a particle perpendicular to **B** during the time t is

$$\Delta x \approx \rho \sqrt{t/\tau} \tag{48}$$

At the same time, the mean displacement during this diffusion process is of the order of $\sqrt{D_{\perp} t}$. Hence, we obtain

$$D_{\perp} \sim \rho^2/\tau \sim \lambda v/(\omega T)^2 \tag{49}$$

The mean time between two successive collisions is proportional to $T^{3/2}$ and therefore we find

$$D_{\perp} \approx An/B^2 \sqrt{T} \tag{50}$$

For high values of B and T, the coefficient $D_{\perp}$ must be very small. But the motion of particles along the field lines is he same as when $B = 0$. Thus, the plasma in a high magnetic field has a sharply marked anisotropy with respect to diffusion.

The above qualitative discussion fails to account for one important feature of the diffusion mechanism. Rigorous theoretical analysis shows that the transverse diffusion of plasma particles is due only to collisions between different particle species, that is, to collisions between ions and electrons. The collisions between the same species cannot give rise to a macroscopic variation of the concentration profile.

Rigorous analysis yields the following expression for the diffusion coefficient:

$$D_{\perp} \approx \frac{\lambda_{ei}\bar{v}_e}{3} \frac{1}{(\omega_{Be}\tau_{ei})^2} = D_0 \frac{1}{(\omega_{Be}\tau_{ei})^2} \qquad (51)$$

Here v_e is the mean thermal velocity of the plasma electrons, ω_{Be} is the electron Larmor frequency, and D_0 is the diffusion coefficient in the absence of the field.

Clearly, the heat conductivity of plasma in the direction perpendicular to **B** also must sharply decrease with increasing field intensity. In contrast to diffusion which is determined by the collisions between ions and electrons, the heat transfer across the field lines in plasma occurs primarily in the ion-ion collisions (if T_i is not too small compared to T_e). This is explained by the fact that the intensity of heat transfer is determined by the size of the region in which there meet the trajectories of particles with different thermal energies under the conditions of the temperature gradient.

The thermal conductivity coefficient in the direction perpendicular to **B** is proportional to the squared size of this region and this size is of the order of the Larmor radius. Therefore, the ion component is primarily responsible for heat transfer.

The coefficient of ion thermal conductivity in the direction perpendicular to the field lines decreases by a factor of the order of $(\omega_{Bi}\tau_{ii})^2$ compared to the coefficient in the absence of the magnetic field. The coefficient $\varkappa_{\perp}$ of the transverse thermal conductivity for the hydrogen plasma is given by the fol-

lowing expression:

$$\varkappa_{\perp} = 2 \cdot 10^{-16} n^2/(B^2 \sqrt{T_i})$$

The rate of equalization of temperature along the radius in the cylindrical plasma column proves to be much higher than the rate of equalization of concentration (approximately, by a factor of $\sqrt{m_i/m_e}$). Hence, a temperature gradient in the direction perpendicular to **B** in the plasma column must vanish long before the plasma spreads all over the space owing to transverse diffusion.

Low diffusion rates make it possible, in principle, to establish in the magnetic field plasma configurations which are confined to a finite volume of space surrounded by vacuum on all sides and have long lifetimes (of the order of seconds or even tens of seconds).

Let us attempt a macroscopic approach to analyze the behaviour of such confined plasma configurations. The first problem that arises here is as follows. The plasma has a gas-kinetic pressure $p = nk(T_e + T_i)$; therefore, a plasma configuration with a certain spatial distribution of pressure can exist for a long time only if in any plasma element the force due to the gradient of gas-kinetic pressure is balanced by forces of other nature. What are these forces? For instance, if we have a single plasma column with a constant pressure p, what is the force that compensates the pressure at the boundary of the column?

In the light of the above discussion of the motion of charged particles in magnetic fields, the answers

to these questions seem almost self-evident. A confined plasma configuration must be contained by electrodynamic forces which arise due to the diamagnetic behaviour of the plasma in the magnetic field. The plasma is diamagnetic since the Larmor currents of the rotating charged particles generate in each plasma element a magnetic moment directed opposite to the external field. This decreases the magnetic field in plasma and gives rise to a ponderomotive force which compensates the pressure difference between the interior of the plasma and its boundary.

The ponderomotive force action on unit volume of the plasma can be expressed in terms of the density j of the diamagnetic current in the plasma and the magnetic induction $\mathbf{B}$. This force is $[\mathbf{j} \times \mathbf{B}]/c$. The equilibrium condition has the following form:

$$\text{grad } p = [\mathbf{j} \times \mathbf{B}]/c \tag{52}$$

Note that the current density j can be found as the sum of the elementary currents due to rotation of electrons and ions in the magnetic field $\mathbf{B}$.

The mechanism of plasma confinement can be illustrated by the following simple example. Fig. 5 shows a diagram of the Larmor currents in a cylindrical plasma column inside which the pressure p is constant. Under these circumstances, the Larmor currents inside the plasma column cancel out each other so that $j = 0$.

But at the boundary of the column there is a net circular current. The force resulting from interaction of this current with the magnetic field compen-

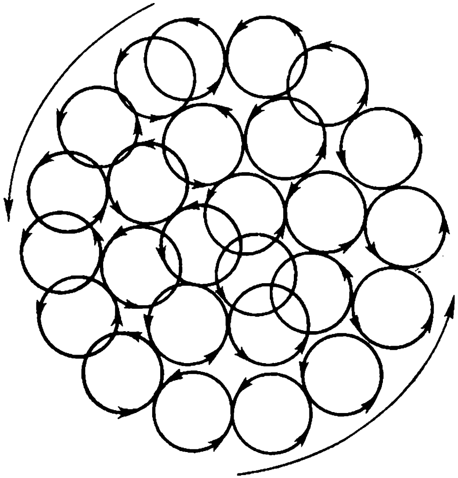

Fig. 5.

Larmor currents in the plasma.

sates the pressure difference at the boundary of the plasma column.

Eq. (52) derived for equilibrium conditions can be extended to analyze the dynamics of plasma configurations. The second Newton's law for the motion of a unit plasma volume can be written as

$$\rho d\mathbf{v}/dt = [\mathbf{j} \times \mathbf{B}]/c - \mathrm{grad}\, p \tag{53}$$

where ρ is the plasma density and $\mathbf{v}$ is the velocity of the plasma volume. The right-hand member of the equation is the force acting on unit plasma volume.

Eq. (53) lacks all characteristics of the plasma as an ionized gas. This equation is generally known as the fundamental equation of magnetic hydrodynamics. It can be used for analyzing the processes which occur in any conducting medium in a strong magnetic field when the medium is capable of changing its form under the effect of external forces, for instance, for analyzing the phenomena in a high-conductivity fluid. In plasma physics Eqs. (52) and (53) give an approximate model of the physical process which is rather rough. However, this model yields very good results in the analysis of statics and dynamics of plasma configurations if we ignore some finer features of the processes and some aspects of the dissipation phenomena in the plasma.

The next approximation is given by a two-fluid model of plasma which accounts for the electron and ion components of the plasma. The following equations describe the dynamics of plasma processes in the two-fluid model:

$$\rho_e \frac{d\mathbf{v}_e}{dt} = -n_e\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_e \times \mathbf{B}]\right) - \operatorname{grad} p_e - \mathbf{F}_{ei} \quad (54)$$

$$\rho_i \frac{d\mathbf{v}_i}{dt} = n_e\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_i \times \mathbf{B}]\right) - \operatorname{grad} p_i + \mathbf{F}_{ei} \quad (55)$$

where ρ_e and ρ_i are the densities of the electron and ion components, $\mathbf{v}_e$ and $\mathbf{v}_i$ are the averaged velocities of electrons and ions, $\mathbf{E}$ is the electric field intensity, and $\mathbf{F}_{ei}$ is the friction force acting on the electrons in unit volume due to their collisions with ions. This force is given by the momentum

transferred per unit time from electrons to ions:

$$\mathbf{F}_{ei}=\frac{\rho_e(\mathbf{v}_e-\mathbf{v}_i)}{\tau_{ei}} \tag{56}$$

The physical meaning of Eqs. (54) and (55) is quite clear as the right-hand members of these equations are the net forces acting on the given particle species in unit volume of the plasma.

The two-fluid model of plasma can be used to analyze such processes as electric current in a magnetized plasma and the development of a variety of non-stationary plasma phenomena. It should be emphasized that both Eq. (53) and Eqs. (54) and (55) are valid only when the plasma pressure is isotropic so that p is a scalar quantity. This condition is satisfied only for such plasma formations in which plasma can freely spread along the lines of force of the magnetic field.

6.

RESULTS DERIVED FROM MAGNETOHYDRODYNAMIC EQUATIONS

Let us now consider some results which can be derived from general magnetohydrodynamic equations. A simple case to analyze is the equilibrium of a plasma column in the longitudinal magnetic field (see Fig. 4). The field lines here are straight and parallel to each other so that the field intensity has only one component, which we denote

as B_z. The intensity B_z is a function of the coordinates x and y which are perpendicular to the field. The equilibrium condition for the plasma column is

$$\operatorname{grad} p = \frac{1}{c}[\mathbf{j} \times \mathbf{B}] = \frac{1}{4\pi}[\operatorname{rot} \mathbf{B} \times \mathbf{B}] \tag{57}$$

Hence we obtain

$$\left.\begin{aligned} \frac{\partial p}{\partial x} &= -\frac{1}{4\pi} B_z \frac{\partial B_z}{\partial x} = -\frac{1}{8\pi} \frac{\partial}{\partial x} B_z^2; \\ \frac{\partial p}{\partial y} &= \frac{1}{4\pi} B_z \frac{\partial B_z}{\partial y} = -\frac{1}{8\pi} \frac{\partial}{\partial y} B_z^2 \end{aligned}\right\} \tag{58}$$

so that

$$p + B^2/8\pi = \text{const} \tag{59}$$

Eq. (59) shows that the magnetic pressure $B^2/8\pi$ outside the plasma is higher than inside it by p. The maximum pressure at which the plasma can be confined by the field of the given intensity B_0 is found from the condition

$$p_{max} = B_0^2/8\pi \tag{60}$$

In this case the plasma must completely push out the field from the volume occupied by the plasma.

If the field lines are curved, the sum of gas-kinetic and magnetic pressures is not generally conserved. However, Eq. (60) regarded as the boundary condition remains valid. If the pressure at the curved surface of the plasma configuration drops to zero, then the following condition must be satisfied:

$$p + B_i^2/8\pi = B_0^2/8\pi$$

Here B_i and B_0 are the field intensities at both sides of the boundary.

A high longitudinal current passing through a plasma can produce a quasi-stationary cylindrical plasma column. In this case the plasma pressure is contained by the magnetic field whose circular lines surround the plasma column. Let us limit ourselves here to the simplest case when current flows only along the thin surface layer of the plasma (strong skin effect). In this case the field intensity inside the plasma is zero and the pressure in it is constant. This pressure can be found from Eq. (60), where B_0 is the intensity of the field generated by the current at the outer surface of the plasma. If the current is I (in the CGSE units) and the plasma radius is a, Eq. (60) yields

$$p = \frac{1}{8\pi}\left(\frac{2I}{ca}\right)^2 = \frac{I^2}{2\pi a^2 c^2} \tag{61}$$

Further, let us assume that the ion and electron temperatures in the plasma column are the same (T). We have here

$$p = 2nkT \tag{62}$$

Eqs. (61) and (62) yield the following relationship between the temperature and the current:

$$T = I^2/4c^2NkT \tag{63}$$

Here $N = \pi n a^2$ is the number of particles of the same species per 1 cm of the length of the plasma column. Eq. (63) has been derived with a special assumption that the current flows in a thin surface layer. But a more careful analysis shows that this formula is valid for any current distribution over the cross-section of the plasma column.

The formation of a plasma column stabilized by the magnetic field of the current flowing in it is known as the linear pinch effect. A current passing through a straight discharge tube filled with gas should, at first, give rise to a plasma (owing to ionization of the gas) which will be compressed by electrodynamic forces so that a plasma column can be formed. In these circumstances the current does three things at once — it generates a plasma, heats it by the Joule heat and compensates its pressure in the compressed plasma column by the pressure of its own magnetic field. At the early stages of high-temperature plasma physics, this technique for generating high-temperature plasmas seemed to be highly promising owing to its apparent simplicity. It was thought that a sufficiently high pulsed current passing through a gas at low pressure can produce in a discharge tube over a negligibly short period of time a plasma column with a tremendous temperature.

Indeed, assume that a current of 10^6 A passes in a discharge tube with a 10 cm diameter filled with hydrogen at the initial pressure of 0.01 mm Hg (the initial concentration of atoms $n_0 = 7\cdot 10^{15}$ cm^{-3}). If all the gas is ionized and a plasma column is formed whose pressure is compensated by electrodynamic forces, the plasma temperature, according to Eq. (63), must be

$$T = \frac{1}{400}\,\frac{I_A^2}{Nk} = \frac{1}{400}\,\frac{I^2}{\pi a^2 n_0 k} \approx 3\cdot 10^7\mathrm{K}$$

In practice, however, the highest temperature obtained with this technique was only about 10^6 degrees

(and that was only for a few microseconds). Experiments have shown that a high-current short-duration pulsed discharge does not generate a quasi-stationary state described by the equilibrium condition (57). Now it can be seen that we could have expected that if we had a keener insight.

At the initial stage of discharge, the current flows only in a comparatively thin layer of the ionized gas near the walls of the discharge tube owing to the skin effect. Under these circumstances, the electrodynamic forces are not compensated by the gas-kinetic pressure of the plasma, since the cylindrical layer with the current surrounds the practically unperturbed weakly ionized gas. Then the plasma layer will be accelerated by electrodynamic forces towards the axis of the tube. This stage of the process can be described very roughly by Eq. (53) without the term grad p. The accelerated motion of the plasma towards the axis of the tube can be regarded as the development of a cylindrical compression shock wave. As the plasma layer travels towards the axis, its mass steadily increases and new gas layers before the plasma front become involved in the compression process. At the same time the gas becomes ionized. At the last stage of compression, the plasma accelerated by the magnetic pressure reaches the axis of the tube. At this stage, a considerable part of the kinetic energy of plasma converts into heat. Now it seems to be the time for the next stage of the process at which the balance can finally be established between the electro-

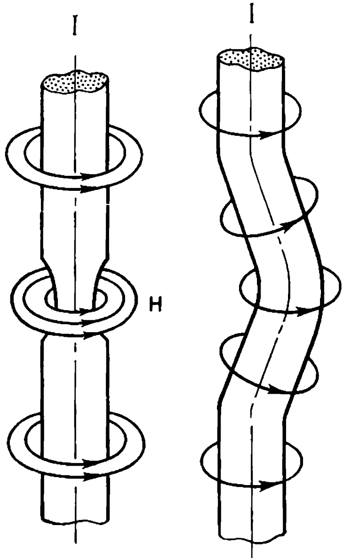

Fig. 6.
Deformation of a plasma column carrying current.

dynamic forces and the internal pressure of the hot plasma. However, here comes into play another physical factor, namely, the instability of the plasma pinch. Under typical experimental conditions deformations of the "bottleneck" and "snake" types develop in the plasma pinch in a few microseconds (see Fig. 6). These deformations disrupt the regular geometrical structure of the plasma so that

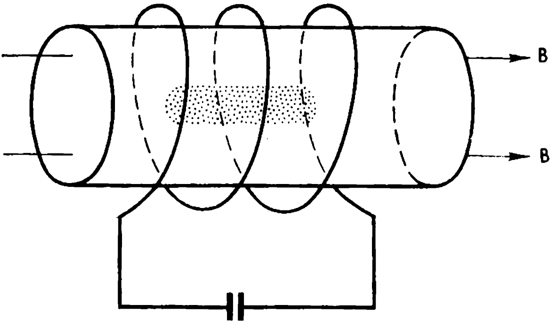

Fig. 7.

Compression of plasma by
a rapidly increasing magnetic field.

it starts to interact strongly with the walls of the discharge tube and rapidly cools down.

This case illustrates for us for the first time a major problem of modern plasma physics, namely, the stability of plasma configurations. The plasma is very unstable in its behaviour; it tries in all possible ways to get rid of the magnetic confinement slipping through the lines of the surrounding magnetic field. At the end of the book we shall discuss the general factors which determine stability or instability of plasma configurations.

A rapidly increasing longitudinal magnetic field compressing the plasma provides for more effective heating of it than in the case of a linear pinch (see Fig. 7). This process also starts with the initial

non-stationary compression stage at which a shock wave travelling to the axis is generated. But when the plasma cylinder radius ceases to oscillate, a balance of forces is established for some time and adiabatic heating of plasma occurs with further increase of magnetic pressure.

A magnetic field which increases up to about 100 kOe in a few microseconds makes it possible to produce a short-lifetime plasma with a temperature about 10^7 degrees and a concentration over 10^{16} cm^{-3}. The longer the discharge tube and the coil which generates the field the longer the lifetime of the high-temperature plasma. But as the plasma can flow out of the open ends along the field lines, the lifetime of high-temperature plasma is still short — for the above values of n and T it cannot be more than a few tens of microseconds in a system about 1 m long.

7.

DEVELOPMENT OF EXPERIMENTAL TARGET

The fundamental problem in the experimental physics of high-temperature plasma is to generate a medium to be studied. Serious experimental studies of the main plasma phenomena can be feasible only when there have been developed the techniques for generating high-temperature plasma whose quasistationary state can exist for relatively long periods. In recent years, some advances have been

made in development of such techniques. There are two main fields of research here. The first one is concerned with magnetic systems which make it possible to generate closed plasma configurations of a toroidal shape. Such machines are often called the toroidal magnetic traps. The second one is concerned with the systems in which the high-temperature plasma is confined in open mirror-type magnetic traps. In such traps the magnetic field increases along the field lines in both directions away from the plasma region. As discussed above the higher-field regions act as mirrors which reflect ions and electrons of the plasma.

Let us discuss at first closed plasma configurations. In such a configuration the plasma spreads freely along the field lines. This means that the field lines must lie within the closed surfaces which are inside the chamber in the region where plasma is generated. A gas-kinetic pressure p of the plasma is a scalar. When the forces acting on the plasma are balanced, the pressure must be constant along the field line since as shown by Eq. (52) the pressure gradient is perpendicular to **B**. Superficially, there would seem to be a very simple technique for magnetic confinement of the circular plasma pinch. This technique consists in winding a regular coil on the surface of a round toroidal chamber which generates a field with circular field lines (Fig. 8). This is the field that will confine the plasma which will be generated inside the chamber in a certain way.

However, this technique is essentially defective as can readily be shown by a simple theoretical argu-

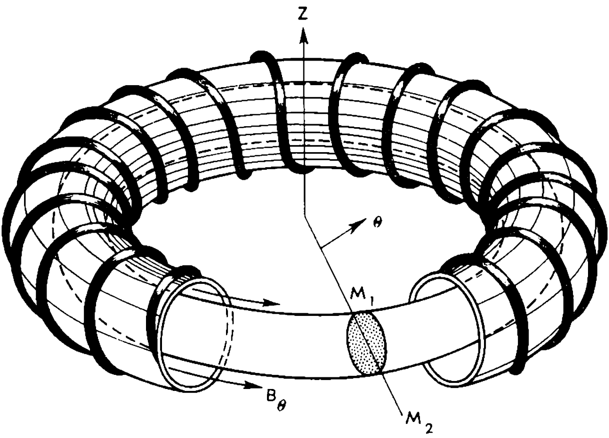

Fig. 8
Plasma ring in the field of a toroidal coil.

ment. The magnetic field in this case is not uniform, since $B \sim 1/R$ so that ions and electrons of plasma will drift perpendicular to the field lines to the chamber walls. The balance equation (52) also shows that plasma cannot be confined in a simple toroidal field. To support this assertion, let us discuss a special case.

Let the plasma completely push out the magnetic field from the region it occupies. Then the following condition must be satisfied everywhere at the boundary of the plasma ring—$p = B^2/8\pi$. But the field is different at the points M_1 and M_2 (see Fig. 8)

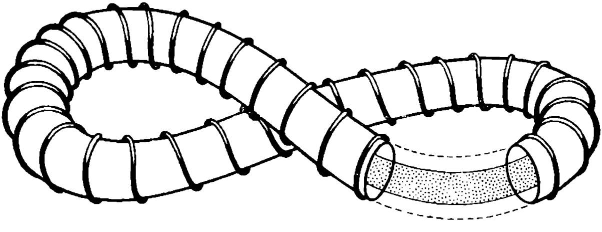

Fig. 9.
A figure 8 configuration.

so that balance is impossible. It can be seen from the balance Eq. (52) that this is so in the general case when $p \neq B^2/8\pi$. The above simple technique for confinement of a circular plasma pinch can be modified in several ways. One modification is to make a figure 8 out of a toroidal chamber with the magnetic coil on it as shown in Fig. 9. Then the plasma pinch also has the figure-8 shape and the drift of particles has opposite directions at the opposite curved parts of the pinch, so that the displacement due to drift during one cycle will be cancelled out. Thus, the laws of motion of the plasma particles in the toroidal "figure 8" do not preclude equilibrium of the plasma. The results of the macroscopic analysis of equilibrium using Eq. (52) should be expected to show that the systems of this type can be utilized as magnetic traps for closed plasma rings.

The toroidal figure 8 was, in fact, the first step in the development of a wide variety of magnetic

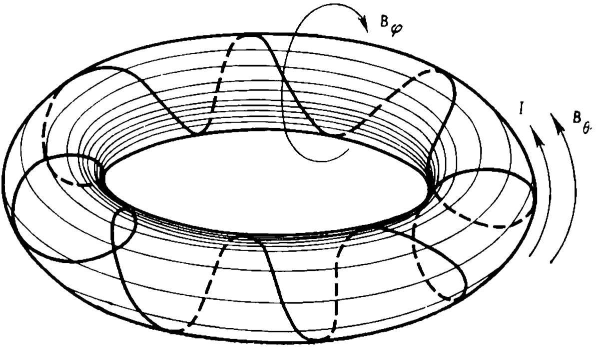

Fig. 10.
Superposition of the fields B_θ and B_φ.

systems for generating closed plasma configurations. These systems based on an elegant concept of Spitzer are known as stellarators. In stellarators the equilibrium of high-temperature plasma is maintained by using external magnetic fields.

The stellarator studies have been for many years an important field of the high-temperature plasma physics. However, we cannot discuss here this very interesting work because this would lead us away from plasma physics towards the analysis of geometry of complex magnetic fields.

Equilibrium toroidal configuration of another type can be generated using superposition of two fields (Fig. 10). One of the fields is produced by a toroidal coil (B_0) and the other field B_φ is produ-

ced by longitudinal circular current flowing in the plasma itself. The field in this system has simple helical lines which wind about the axis of the plasma pinch where the magnetic field of the current vanishes. The drift displacements of particles are compensated owing to the curvature of the field lines. To understand the mechanism of this compensation without calculations, we shall illustrate it by a special case when the following two conditions are satisfied: (1) the magnetic field generated by the current in the plasma is low compared with the toroidal field generated by the external coils; and (2) a plasma particle has a sufficiently high longitudinal velocity and therefore can freely travel along the field line without being reflected from the higher-field regions. Under these circumstances the motion of the centre of the Larmor circle is the superposition of two motions—the motion along the field line (always in the same direction) and the drift due to non-uniformity of the toroidal magnetic field. Fig. 11 illustrates a simple model for superposition of these two motions in which the continuous drift along the axis y (the principal axis of the toroidal system) is expressed by two jump-like displacements. We see that the motion of the particle along the winding field line results in closing of the projection of its trajectory on the cross section of the plasma pinch so that the net drift displacement is zero.

Analysis of the motion of particles in the superimposed fields of the above type shows that drift displacements are compensated not only for the

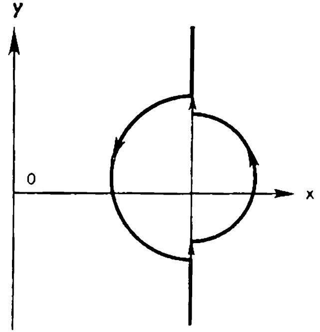

Fig. 11.
Model of the drift motion of particles in a helical field.

particles with high longitudinal velocities which freely travel along the helical field lines, (the so-called "through-going" particles) but also for the so-called "confined" particles which oscillate along the field lines owing to the reflection from the higher-field regions.

It should be stressed here that the compensation of drift displacement does not preclude any effect of the drift motion on the shape of the trajectory. Even the schematic model of Fig. 11 shows that the drift motion results in some deformation of the trajectory. This deformation is even more strongly marked for the trajectories of the confined particles. We shall discuss some of the resulting effects below when mentioning the problems of the rates of diffusion and heat transfer.

8.

CONDITIONS FOR EXISTENCE OF A PLASMA RING

Let us now discuss in more detail the conditions for the existence of a plasma ring in a longitudinal external magnetic field and the magnetic field generated by the current flowing in the ring. We illustrate the general problem by analyzing in detail this special case because, first, this is the simplest example of a magnetic confinement and, second, the longest times of high-temperature plasma confinement have been obtained in the experiments with the machines utilizing this concept.

Before we analyze the forces acting on the plasma ring, let us look at some of the characteristics of the magnetic field geometry in the toroidal trap under consideration. As mentioned above, the helical field lines wind about the circular axis of the plasma ring. Let the plane S be the cross section of the trap. The field line through the point M_1 (Fig. 12) in this plane after passing around the toroidal magnetic system will intersect the plane S at the point M_2, next time it will intersect it at the point M_3, etc. The set of points M_1, M_2, M_3, ... is, generally, infinite. But for some field lines this set can be finite so that they are closed by themselves. Such lines are known as "degenerate" lines. The axis of the ring at which the field is zero is such a degenerate line; it is shown by point O in Fig. 12.

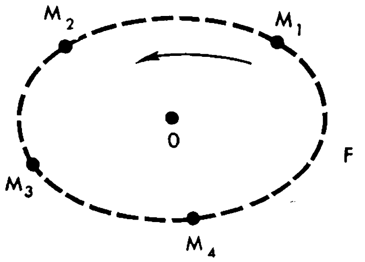

Fig. 12.
Rotation of a field line (rotational transformation).

The behaviour of the field line which winds repeatedly about the toroid is determined by the positions of the "descriptive" points M_1, M_2, M_3, ... Assume that the point M_n after n cycles is close to the initial point M_1. Then after the next cycle the angle of rotation about O will be more than 360°. Where will be the next point? It is natural to suggest that the point M_{n+1} will be between the points M_1 and M_2, the point M_{n+2} between the descriptive points M_2 and M_3 and so on. Thus, after numerous cycles, the points in the cross section of the trap will make up a smooth closed curve (curve F in Fig. 12). Thus, we can say here that there is a toroidal magnetic surface which, so to say, consists of (or is tightly woven of) one infinite field line.

Analysis of the equations for a magnetic field supports the assumption about magnetic surfaces

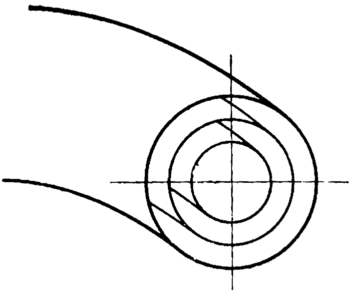

Fig. 13.
Magnetic surfaces in a helical field.

in the system with the external toroidal field B_θ and the field B_φ of the plasma current. As shown by the analytical results, for regular distribution of the current density over the cross section of the plasma ring the sufficient general condition for the existence of magnetic surfaces is the axial symmetry of the field. When the axial symmetry exists, the equations for the magnetic field lines can be integrated yielding the analytical expression for magnetic surfaces. The magnetic surfaces comprise the set of embedded toroids (see Fig. 13). Any toroid out of this continuum is comprised of one field line. However, this set of magnetic surfaces includes a countable set of "degenerate" surfaces generated by closed field lines. In this case the surface is woven of a continuous set of field lines shifted with respect to each other.

Spitzer has suggested that magnetic surfaces exist also for such toroidal fields in which the helical field lines winding about the circular axis are generated by the external current sources using special helical windings or by deforming the whole magnetic system into, for instance, a figure 8.

However, theoretical analysis shows that for such asymmetrical fields the concept of embedded magnetic surfaces can be used only to some approximation. The structure of the magnetic field is, generally, rather complicated. The individual magnetic surfaces are separated by a system of toroidal tubes ("filaments") between which the behaviour of the field lines is quite chaotic. Clearly, the existence of magnetic surfaces is very important since it determines the prospects for utilization of magnetic systems of various types for confinement of hot plasma. Since the plasma freely spreads along the field lines, its pressure p must be the same at different points of the same magnetic surface. Thus, magnetic surfaces are families of plasma isobars. If the points M_1, M_2, M_3, ... fill up a certain wide region in the cross section of the plasma ring instead of comprising a smooth curve, then the plasma within this region can be at equilibrium only if grad $p = 0$. The fact that the pressure gradient vanishes at the cross section of the plasma ring is, clearly, equivalent to the development of abnormally high diffusion in the direction perpendicular to **B**.

Let us now return to the analysis of the equilibrium conditions for a circular plasma ring proceed-

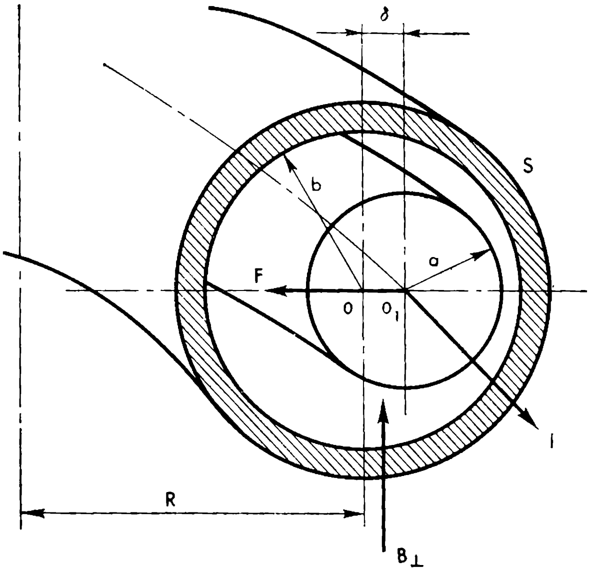

Fig. 14.
Cross section of a plasma ring.

ing from the equations of magnetic hydrodynamics. In the first approximation the cross section of the plasma ring can be assumed to be the circle of radius a. The plasma pressure and the current density vanish outside the circle of radius a. The toroidal surface of the radius a is the boundary surface of the plasma ring which has no contact with the walls of the vacuum chamber in which it has been

generated (Fig. 14). This surface is also one of the magnetic surfaces of the toroidal trap. Another fundamental geometric parameter of the system is the major radius R of the plasma ring. Below we shall always assume that a is much less than R as is invariably the case in experimental environments. Note that when the ratio a/R is small, the characteristics of the plasma ring are close to those of an infinite plasma cylinder. Since the problem has two fundamental geometric parameters a and R for two degress of freedom of the plasma ring, we have to find two equilibrium conditions—for the minor radius and for the major radius. For the first one we can neglect the effect of the toroidal structure for the minor radius. Thus, we can use the equilibrium equation for the plasma cylinder. Since the plasma in this case is under the effect of two fields, the equilibrium equations can be synthesized from Eqs. (61) and (62) which were derived for either the longitudinal field or the field of the current. We can derive such equilibrium equations for the plasma cylinder assuming that both the fields and the plasma pressure depend only on r, that is, on the distance between the given point and the axis of the plasma cylinder.

It can be easily shown that with the above condition, equilibrium necessitates the identity between the sum of the plasma pressure p and the pressure $B_\theta^2(r)/8\pi$ of the longitudinal magnetic field in the plasma averaged over the cross section of the plasma column and the sum of the pressures of the longitudinal field and the field of the current

at the boundary of the plasma column:

$$p + B_\theta^2(r)/8\pi = B_\theta^2(a)/8\pi + B_\varphi^2(a)/8\pi \qquad (64)$$

Eq. (64) can also be written in the following form:

$$2\pi a^2 \bar{p} = I^2/c^2 + [B_\theta^2(a) - \overline{B_\theta^2(r)}]/4 \qquad (65)$$

It can be demonstrated by detailed theoretical analysis that Eq. (65) is also valid when we take into consideration in the first approximation the toroidal structure of the system. The resulting equilibrium condition differs only by the corrected parameter $B_\theta(a)$, which, under rigorous treatment, is the longitudinal field at the surface of the plasma for $\varphi = \pi/2$ (see Fig. 14).

To derive the equilibrium condition for the major radius, we have to analyze the character and magnitude of forces which can result in variation of R. Let us restrict ourselves here to a rough qualitative analysis of these forces. There are three various factors giving rise to such forces.

(1) *Electrodynamic Stretching of the Ring Current.* According to the fundamental laws of electrodynamics, the ponderomotive force acting on a conductor carrying current always has such a direction that it tends to increase its inductance. In our special case the inductance increases with the major radius R.

Thus, we encounter here tensile stresses. This is almost self-evident, though. Since the ponderomotive force is due to the interaction between the current and its magnetic field it is proportional to I^2 for given geometric parameters.

(2) *Stretching of the Toroidal Ring under the Action of the Internal Pressure.* To find the tensile force due to the above factor, we have to calculate the work performed by the plasma pressure for infinitesimal variation of R and to divide the result by δR. The total tensile force proves to be proportional to $a^2 p$.

(3) *Radial Ponderomotive Force due to the Difference Between the Intensities of the Longitudinal Magnetic Field Inside and Outside the Plasma.* When $B_\theta^2(a)$ is higher than $B_\varphi^2(a)$, this force is directed towards increasing R.

The evaluation of the net effect of the above forces taking into account the equilibrium conditions for the minor radius shows that the net effect is to increase the radius R, that is, the pinch carrying current must stretch. This means the lack of equilibrium. But one step more and we reach our goal. The tensile force can be compensated by applying the magnetic field $B_\perp$ perpendicular to the equatorial plane of the plasma ring (see Fig. 14). If we have appropriately chosen the direction of $B_\perp$, it will act with the force $2\pi RIB_\perp/c$ on the ring carrying the current I (referred to the total length of the ring). This force can compensate the net effect of the tensile forces discussed above.

In the experimental machines in which toroidal plasma rings are generated, the tensile forces are compensated automatically, since the vacuum chamber has a thick conducting (metal) body. A displacement of the centre of the cross section of the plasma ring with respect to the centre of the cross

section of the toroidal conducting chamber body gives rise to eddy currents in the body. Eddy currents generate the transverse magnetic field which compensates the tensile forces. It can readily be shown that the compensating compressive force is proportional to I^2 and the displacement δ. Hence, equilibrium is reached at a certain value of δ.

V. Shafranov has analyzed this equilibrium problem theoretically and found the equilibrium displacement δ*:

$$\delta = \frac{b^2}{2R}\left\{\ln\frac{b}{a} + \left(1-\frac{a^2}{p^2}\right)\left[\frac{8\pi p}{B_\varphi^2(a)} + \frac{L_i - 1}{2}\right]\right\} \qquad (66)$$

Here b is the radius of the cross section of the conducting body, $B_\varphi(a)$ is the magnetic field generated by the current at the boundary of the plasma ring, and L_i is the internal inductance of the plasma ring per unit length. Eq. (66) determines the position of the boundary magnetic surface of the plasma ring. Since the system is toroidal, the cross sections of the magnetic surfaces inside the plasma ring must not be represented by concentric circles (as is done for the plasma cylinder). Plasma carrying longitudinal current which is inside any of the internal magnetic surfaces has the same equilibrium conditions with respect to the outer plasma layers as the conditions discussed above for the plasma ring surrounded by the conducting chamber

* A rigorous definition of δ is the distance between the geometric centre of the cross section of the plasma ring and the geometrical centre of the cross section of the body.

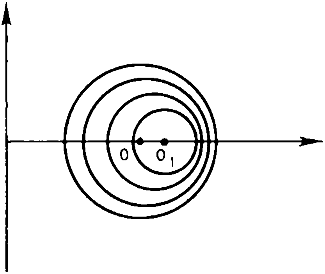

Fig. 15.
Displacement of magnetic surfaces in a torus.

body. Therefore, we expected the centres of the circles representing the cross sections of the magnetic surfaces to be displaced outwards, towards the increasing R the larger, the smaller their radiuses (see Fig. 15). The largest displacement with respect to the centre of the cross section of the conducting body corresponds to the magnetic axis of the system where the current field B_φ vanishes. These are the main results of theoretical analysis of equilibrium of the plasma rings carrying current.

Let us note an interesting fact. The longitudinal magnetic field does not have any effect on Eq. (66) which determines the equilibrium position of the plasma ring in the chamber. It is as if this field is non-existent. It seems that logically the next step is to get rid of the longitudinal magnetic field

in the ring system altogether. This is quite admissible in the framework of the equilibrium theory. But once we become concerned with the stability of the plasma ring, the longitudinal field becomes again indispensable.

When there is no longitudinal magnetic field, the bending and bottleneck-type deformations can develop without limitations so that in a very short time (of the order of a/v_i, where v_i is the thermal velocity of ions) the regular shape of the plasma ring will spread out and the plasma will rapidly cool down owing to intensive interaction with the chamber walls. A strong longitudinal magnetic field produces a sort of a rigid frame of the field lines which stabilizes the plasma ring. Thus, even though the longitudinal field is indifferent with respect to the equilibrium conditions, it is instrumental in making the plasma ring stable. This point, clearly, has to be discussed in more detail as is done below.

Plasma is a good conductor; hence, when it travels rapidly in a magnetic field and crosses its lines, induced currents appear which change the distribution of the intensity of the magnetic field. The general effect of this change is as if the plasma in its motion carries with it the field lines which behave as if they were "frozen" into the plasma or "glued" to it. To get a better understanding of the "freezing" of the field lines, let us consider the following special case of plasma motion.

Let a magnetic field parallel to the axis z be applied to a volume of plasma. The field varies as

a given function in the plane xy perpendicular to the axis z. The plasma density is constant along the field lines but can vary with the coordinates x and y. Assume that the plasma travels perpendicular to the field lines. Let us consider a small plasma column parallel to **B** whose cross section in the plane xy is dS. During the motion of plasma, this plasma column will be compressed or stretched so that its cross section will be changed. Since we deal here with rapid processes when diffusion can be neglected, the total number of particles in the column must remain constant. Hence, we obtain

$$n dS = \text{const} \tag{67}$$

On the other hand, the magnetic flux through the cross section dS must remain constant, too, if the motion is rapid. If the plasma conductivity is high enough, even a slight variation of the magnetic flux, if it occurs rapidly, gives rise to a high induced current. The magnetic field of this current compensates variation of the magnetic flux. Conservation of the flux means that

$$B dS = \text{const} \tag{68}$$

Comparison of Eqs. (67) and (68) shows that under these circumstance the ratio B/n remains constant with the motion of plasma. If the plasma is compressed, the magnetic field in it increases and if plasma is expanded, the field decreases. Redistribution of the plasma density due to rapid deformations results in appropriate variation of spatial distribution of the magnetic field. The "freezing"

of the field lines is just the effect of this relationship between deformations of plasma and variations of the field which follows from conservation of the magnetic flux in a travelling volume element of the plasma.

It should be noted, however, that the ratio B/n is not always constant; this is so only in the specific case discussed above. In different geometric conditions the relationship between the magnetic field and the density in the rapidly travelling plasma can differ from the above one. For instance, in an axially symmetric plasma jet travelling under the effect of electrodynamic forces due to current in the plasma, the parameter that remains constant for each plasma element during the motion is B/nr, where r is the distance between this element and the axis of the system.

There is another noteworthy point. The "freezing" of field lines is not an effect which is unique to a plasma. A similar effect can be found in any good conductor travelling in a magnetic field with high enough speed.

The stabilizing effect of the longitudinal field on the plasma ring is due to the "freezing" of field lines into the plasma. If a part of the ring gets more narrow, the longitudinal field in this part increases owing to conservation of its flux; hence, the magnetic pressure in the plasma in this part also increases.

This force prevents the development of deformation. Bending deformations result in elongation of the "frozen" lines of the longitudinal field and hence in increase of the Maxwellian stresses. This effect

can also prevent the development of deformations.

The effect of the longitudinal magnetic field on some types of deformations of the plasma ring is a special case of those effects which are analyzed by the general theory of stability of plasma configurations. Later we shall discuss some of the general aspects of the theory but until that we shall sometimes note some results obtained by applying the theory of stability to the specific problems we are interested in.

9.

THE THEORY OF STABILITY

This theory can be applied, for instance, to selecting a particular technique for confinement and thermal insulation of the plasma ring using the combination of the longitudinal magnetic field and the field of the plasma current. As predicted by the theory, the system has the highest stability with respect to the most dangerous large-scale deformations when the longitudinal magnetic field B_θ is much higher than the current field B_φ. The stability condition can be written in the following form:

$$(B_\theta/B_\varphi)\,(r/R) > 1 \qquad (69)$$

Here r is the distance from a given point inside the pinch to the magnetic axis at which the field B_φ vanishes. The stability condition (69) has been derived independently by M. D. Kruskal and V. D. Shafranov. Condition (69) must be satisfied

throughout the plasma ring. If we take into account the character of distribution of the current density over the cross section of the plasma ring (this distribution is always bell-shaped), we find that inequality (69) is satisfied at any point of the plasma if it is satisfied with a margin at the plasma boundary, that is, for $r = a$. Hence, the Kruskal-Shafranov condition can be written in the following form:

$$(B_\theta/B_\varphi)\,(a/R) \geqslant q \qquad (70)$$

The quantity q is known as the margin of stability. It seems that stability is good enough when q is more than 3 unless the current distribution is very unusual.

For a fairly long time, the experimenters in the USSR have been studying toroidal systems, where plasma is confined by the magnetic field of the current circulating in the plasma and the plasma ring is stabilized by a very high longitudinal field. Machines of this type are known as "Tokamaks". Fig. 16 shows schematically a tokamak-type installation. The toroidal discharge chamber encircles the core of the transformer which induces current in the plasma ring. The chamber has an inner shell made of thin-sheet stainless steel and an outer shell of thick-sheet copper. There is an electric insulation between two shells and the space between them is evacuated. Before each experiment, long-time heating is used to degas the inner shell, the so-called "liner", inside which the plasma is generated by the induced electric discharge. The outer copper shell stabilizes the plasma

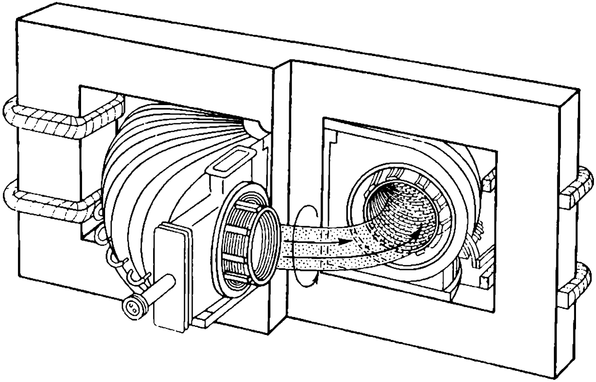

Fig. 16.
General view of a Tokamak.

by eddy currents generated with displacement of the plasma ring. The longitudinal field B_θ is generated by the magnetic coils installed on the chamber. In a medium-scale Tokamak (T-4), the radius a of the cross section of the plasma ring can be as high as 15-18 cm for $R = 100$ cm (the maximum value of a is determined by the size of the diaphragm inside the liner).

The experiments with the T-4 Tokamak were carried out with the highest longitudinal field of about 37 kOe and the current density varied from 100 to 200 A/cm^2. The duration of the current pulse could be varied from a few milliseconds to 50-60 ms. The mean plasma density in these expe-

riments was varied from $n_e \approx 1 \cdot 10^{13}$ to $4 \cdot 10^{13}$ cm^{-3} by changing the initial gas pressure in the chamber. The experiments with other Tokamaks were carried out in a wider density range (from $2 \cdot 10^{12}$ to $1 \cdot 10^{14}$ cm^{-3}). All the principal experiments were done with hydrogen and deuterium.

The plasma in Tokamaks is heated naturally due to the Joule heating of the current flowing in it. Under these circumstances, the current density in the plasma is the principal factor determining the heating of plasma. The Kruskal-Shafranov stability condition shows that the permissible current density increases as the longitudinal field. Hence, the higher B_θ the higher the plasma temperature th it can be obtained.

The experimental results from Tokamaks fully support the validity of the Kruskal-Shafranov condition. The results show that if the stability margin q is sufficiently large (over 3 or 4) all the signs of the large-scale instabilities disappear and the plasma can be heated up to a high temperature. The current experiments with the T-4 machine with a current pulse of 25-60 ms and the longitudinal field of 37 kOe produce the hydrogen plasma with the following optimum parameters: $n_e \approx$ $\approx 4 \cdot 10^{13}$ cm^{-3}, $T_e \approx 1 \cdot 10^7$ degrees, $T_i \approx 5 \cdot 10^6$ degrees. Analysis of the energy balance of the plasma yields energy liberated in the plasma ring owing to the Joule heating and the thermal energy flux from the plasma ring. The magnitude of this flux is a gauge of the efficiency of thermal insulation of plasma which is related to the magnetic field. The

efficiency of thermal insulation can be characterized by the parameter known as the mean time of energy retention in the plasma and is defined in the following way

$$\tau_E = W/Q \tag{71}$$

Here W is the total energy stored in the plasma ring, Q is the energy leaving the plasma per unit time (lost power) and τ_E is the mean time of energy retention. The time τ_E in Tokamaks can be as long as 15 ms under the optimum conditions (high longitudinal field, high plasma current, and high enough plasma density).

The mean lifetimes of fast electrons and ions in the plasma can also be determined in experiments. In the T-4 machine, these lifetimes are more than 20-30 ms under the optimum conditions. An ion travels about 10 km in the chamber during this period and undergoes about a hundred Coulomb collisions. A comparison of the confinement time for particles in Tokamaks and the time predicted from the diffusion theory using the expression for the diffusion coefficient in the magnetic field B_θ given above shows that the experimental confinement time is by a few orders of magnitude smaller than the predicted time. A few years ago this fact was assumed to indicate the existence of "anomalous" diffusion due to development of a "slow" instability of plasma. This idea was supported by the measured heat losses from the plasma ring. These losses were by a few orders of magnitude higher than those predicted by the theory of plasma

heat conductivity in a magnetic field using in the expression for the heat conductivity coefficient the value of B equal to the longitudinal field in a Tokamak.

It would seem that the anomalous heat conductivity can be attributed to the development of "slow" instabilities. We cannot be quite sure, however, that high loss rates of plasma energy and particles in Tokamaks are due to any instabilities. This is because rigorous classical theory of diffusion and heat conductivity shows that the diffusion and heat conductivity coefficients in Tokamaks should be higher by about three orders of magnitude than it was assumed earlier. Intuitively, it can readily be seen from qualitative considerations that the formulas for coefficients of diffusion and heat conductivity for the Tokamak magnetic systems should be modified. Particles in Tokamaks travel in a composite magnetic field and the longitudinal field B_θ alone cannot hold them. Hence, the current field B_φ must play the major part in the transport phenomena and, since $B_\varphi \ll B_\theta$, the transport rates should be expected to be higher than in the uniform field B_θ.

The correct quantitative theory of transport phenomena in toroidal systems was developed in two steps. At first, B. B. Kadomtzev and V. D. Shafranov took into consideration the effect of toroidal drift on the trajectories of particles and found that the coefficients of diffusion and heat conductivity must be increased by a factor of about $2q^2$, where $q = [B_z/B_\varphi(a)]\,(a/R)$ if the plasma density

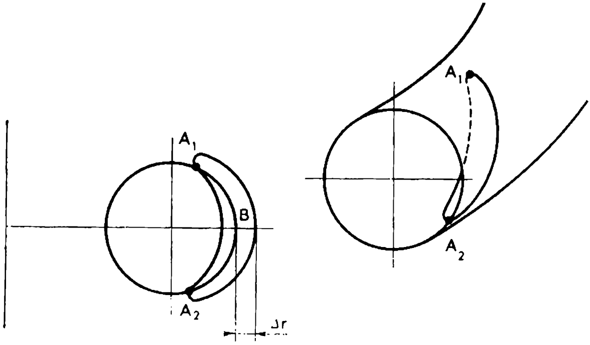

Fig. 17.
Trajectory of a confined particle.

is high enough and there is no physical meaning in classifying particles as through-going and confined. Thus, the diffusion and heat conductivity coefficients under normal conditions of the experiment must be higher by a factor of 20-30.

The next theoretical advance was done by R. Z. Sagdeev and A. A. Galeev. They noticed that a toroidal drift had an especially strong effect on the confined particles which oscillated along the lines of force being reflected from the high-field regions. Fig. 17 shows such a trajectory and its projection on the plane in which the system's axis lies and which rotates by the angle θ with the particle. The particle also oscillates along the radial direction so that the trajectories of particles in the magnetic

field are mixed and the rates of diffusion and heat transfer increase greatly. These rates are higher than the earlier predictions (B. B. Kadomtzev and V. D. Shafranov) by a factor of $(R/a)^{3/2}$, that is, about 30 times under the experimental conditions. This effect occurs when collisions in the plasma are rare enough; therefore, we can classify the particles into confined and through-going ones. In Tokamak experiments, collisions are typically rare, hence, the coefficients of diffusion and heat conductivity must be higher by three orders of magnitude than assumed earlier.

It is yet early to assert that the experimental results are in a very good agreement with the modified classical theory. The current experimental and theoretical accuracies of the values of the diffusion and heat conductivity coefficients are such that if they differ by a factor of two or three, this does not necessarily mean contradiction in the results.

The available results suggest that at least under some experimental conditions when a plasma has a relatively high ion temperature and a not-too-low density, the measured rates of diffusion and heat transfer are close to those predicted by the theory of R. Z. Sagdeev and A. A. Galeev.

However, we still cannot rule out the possibility that the slowly developing so-called "drift" instabilities also contribute to the loss of particles and heat energy from a high-temperature plasma. But even now it is clear that the plasma temperature and the time of energy retention increase conside-

rably with further increase in the magnetic field and the cross section radius of the plasma ring in Tokamaks.

In experiments with toroidal systems, the primary aim remains to be the same—to increase the plasma parameters (temperature, pressure and lifetimes of particles) further. Only when stable high-temperature plasma configurations have been generated, we shall be able to study in earnest the fine physical effects typical of plasma, in particular, those sophisticated tunes that could be readily played with unique resonator.

10.

PLASMA BEHAVIOUR IN TRAPS WITH MAGNETIC MIRRORS

Plasma is confined in such systems owing to reflection of particles from high-field regions. In the magnetic mirror traps the distribution of velocities of plasma particles is non-isotropic.

Let the maximum intensity of the magnetic field along one of the field lines is B_{max}. This value can be assumed to be the same at both ends of the trap since, if the values were different at the ends, the confinement of particles would be determined only by the reflection of particles from the mirror with a lower field.

Let us consider a small segment of the field line at which the field intensity is B. For a long lifetime

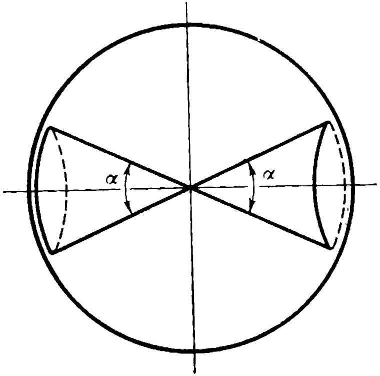

Fig. 18.

Loss cones in a trap with magnetic mirrors.

of the plasma in the trap, only those charged particles, for which the angle α between the velocity and the field line is larger than $\arcsin\alpha = \sqrt{B/B_{max}}$, can be within this segment. All the particles with smaller angles α will leave the trap within a very short period (of the order of l/v_i, where l is the trap's length and v_i is the thermal velocity of ions). In particular, near the magnetic mirrors, where B is close to B_{max} only such particles can be found whose longitudinal velocity is relatively small and the angle α is close to 90°. Fig. 18 presents the sphere of all possible directions of the particle's velocity. The velocity vectors of the particles which freely leave the trap lie within

two cones in this sphere. These cones are called loss cones. In such a trap the plasma pressure, clearly, is not a scalar quantity. We have to distinguish between the longitudinal pressure

$$p_{\|} = n\,(m_i\overline{v_{\|i}^2} + m_e\overline{v_{\|e}^2})$$

and the transverse pressure

$$p_{\perp} = n\,(m_i\overline{v_{\perp i}^2} + m_e\overline{v_{\perp e}^2})/2$$

If the velocity vector of a particle gets into a loss cone owing to a Coulomb collision, this particle will immediately leave the plasma. If the angle of the loss cone is large enough, practically one Coulomb collision is sufficient to expel a confined particle from the trap with magnetic mirrors. Therefore, the mean lifetime of particles in such traps must be fairly close by the order of magnitude to the mean time τ_{ii} between two successive ion-ion collisions. The ion-ion collisions play here a decisive role, since electrons, the collisions between which are more frequent, cannot leave the trap independently of ions. At the moment of plasma generation in the trap, a relatively small number of electrons will leave it so that the plasma will have a positive charge and the flows of particles of both signs will be balanced.

The above results show that even under optimum conditions when the loss of particles is due only to Coulomb collisions the open-ended traps with magnetic mirrors have a much lower confinement times than the toroidal traps in which the mea-

sured confinement times are higher than τ_{ii} by two orders of magnitude. But an important experimental advantage of the open-ended traps is the possibility of using a variety of techniques for generating a high-temperature plasma. In open-ended traps, a plasma can be generated by injection of fast particle flows, capture of plasma jets, high-frequency heating of cold plasma, production of fast ions in cold plasma jet by specially generating instabilities, etc. These traps also make it possible to vary in wide ranges the energy spectra of electrons and ions and to generate a plasma with high-energy ions and low-energy electrons or, on the contrary, with very fast electrons and relatively cold ions.

At an early stage of the experimental open-ended system studies of plasma, the only systems used were those with two magnetic mirrors at the ends. The magnetic field in such traps increases along the field lines in both directions from the central region where the plasma should be. At the same time, the field decreases radially. Numerous experiments have shown that this simplest open-ended trap is not suitable for long confinement of plasma with a hot ion component. For the plasma density over 10^7-10^8 cm^{-3}, the lifetime of plasma in a simple two-mirror trap is not more than a few tens of microseconds. The plasma flows out from the trap owing to the development of the so-called flute instabilities. This instability is due to the fact that the plasma is a diamagnetic and therefore it can easily travel towards a weaker field by producing "tongues" which seep between the field

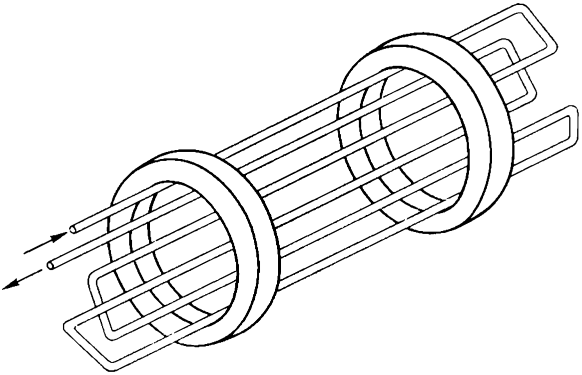

Fig. 19·
Diagram of a magnetic system with a combined field.

lines (between the "tongues" there appear flutes without plasma).

To prevent the development of flute instability, in the simplest magnetic mirror traps such techniques have to be applied which by themselves result in considerable deterioration of thermal insulation of plasma which leads to its cooling (low vacuum, intense low-voltage arc in the space filled with plasma, etc.).

In 1961 M. Ioffe was the first to experiment with an open-ended trap with the magnetic field increasing in all directions from the plasma region. Fig. 19 illustrates the magnetic system of such a trap. The magnetic field is produced by coaxial coils and

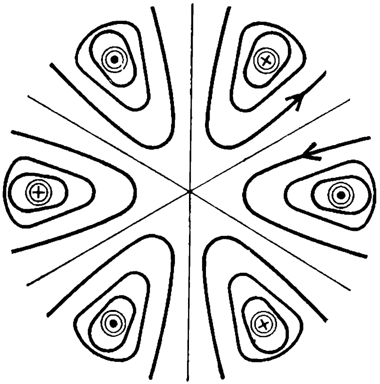

Fig. 20.
Field of linear conductors.

six linear conductors carrying current which are placed symmetrically with respect to the central axis of coils. Currents flowing in the neighbouring conductors have opposite directions. This system without linear conductors would be a simple mirror trap with a radially decreasing field. The linear conductors generate the field whose geometry is shown in Fig. 20. This field varies as r^2, where r is the distance to the symmetry axis. In combined fields of the coils and linear conductors, the resulting field intensity increases in all directions from the central region of the trap where a plasma is generated. This is known as the minimum-B concept.

The experiments with minimum-B machines have shown that they make possible stable confinement of plasma with a density up to about 10^{11} cm^{-3} and the ion energy of the order of 1 keV ($T_i \sim \sim 1 \cdot 10^7$ degrees). The lifetimes of plasma particles in such machines amount to tens of milliseconds. Thus, a technique has been found for suppressing one of the most dangerous rapidly developing instabilities typical of hot plasma. Therefore, all modern open-ended hot-plasma systems are designed proceeding from the minimum-B concept.

It is yet unclear whether higher temperatures and plasma densities in the minimum-B machines can give rise to other types of plasma instability. Theoretical predictions give some food for these thoughts. Analysis of plasma behaviour in open-ended machines with magnetic mirrors shows that the peculiar distribution of particle velocites in such systems gives rise to various mechanisms which generate so-called kinetic instabilities. The kinetic instabilities are due to amplification of the plasma oscillations by directed flows of ions or electrons. These instabilities can readily develop if the velocity distribution function has a maximum at a nonzero velocity. The existence of such a maximum is equivalent to the existence of a directed flow of particles.

If the plasma particles have the Maxwell velocity distribution function (the dotted line in Fig. 21), the oscillations are not amplified. But in an open-ended machine, the velocity distribution function for confined particles must have a distinct maximum

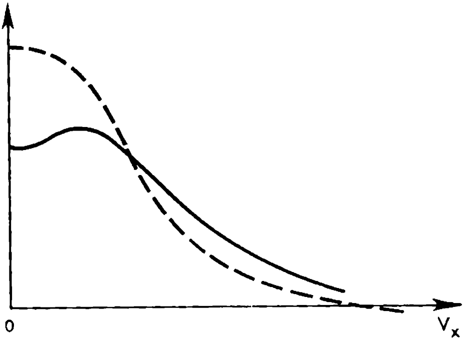

Fig. 21.

Velocity distributions of particles: Maxwell distribution—dashed line; distribution in the presence of a loss cone—solid line.

for the transverse velocity component owing to the existence of the loss cone (see the solid line in Fig. 21). With this shape of velocity distribution, there can occur amplification of ion-electron oscillations resulting in a sharp increase of the plasma loss from the trap.

The first to analyze theoretically this type of instability were Rosenbluth and Post; they have shown that this instability develops when the length of the plasma clot trapped in the machine is more than a few hundreds Larmor radiuses. This instability can, apparently, be stabilized in a short magnetic system.

The theory predicts another type of instability which is due to non-uniform distribution of plasma density in the direction perpendicular to the field (this non-uniformity is always present merely because of the finite cross section of the plasma configuration). The drift motion of ions in non-uniform plasma can give rise to amplification of ion oscillations of a frequency close to the Larmor frequency ω_{Bi}. The oscillations are amplified if the Larmor radius of ions amounts to about $a\,[B^2/(4\pi n m_i c^2)]^{2/3}$, where a is the radius of the plasma.

The experimental plasmas in open-ended machines have the parameters close to values at which the conditions for the development of the above instabilities are satisfied. However, up till now such instabilities have not been observed. Therefore, we can hope that we can attain higher temperatures and densities for stable plasma confined in the minimum-B open-ended machines.

11.

STABILITY OF PLASMA CONFIGURATIONS

Even a very brief discussion of plasma physics cannot ignore the central problem of this science, namely, the stability of plasma configurations. The advances in stability studies determine the prospects for increasing the temperature and lifetime of hot plasma.

Our general discussion of plasma stability will lack mathematical rigour. With this in mind, let us turn to our subject. Let us assume that a confined plasma configuration is suspended in vacuum by a magnetic field and is at magnetohydrodynamic equilibrium. In such a plasma, the temperature and density remain constant and Eq. (52) is satisfied at each point. Will the equilibrium state be preserved for a long time or the random fluctuations in the plasma will generate increasing instabilities and the plasma will rapidly spread out over the available volume? This is the generally formulated stability problem. A significant factor in this problem is that different instability mechanisms produce perturbations with widely different rates of increase in the plasma.

The development of theory should, to a certain extent, take this fact into account. Thus, the origin of instabilities and the techniques for their stabilization should be considered, starting from the instabilities which give rise to rapid macroscopic motion of plasma and are, hence, the most dangerous. When these, the most significant instabilities have been analyzed, we can then analyze less destructive instabilities and so on. It can appear that if we take into consideration all the types of instability of plasma confined in a magnetic field, we shall find that an absolutely stable plasma configuration is unfeasible. But absolute stability is, maybe, not essential. In practice, we must learn how to inhibit the development of various instabilities, so that non-stabilized weak and slow-deve-

loping instabilities would not seriously hinder accumulation and long-time retention of thermal energy in high-temperature plasma.

The most dangerous instabilities are those which give rise to motion of macroscopic plasma elements with the velocities of the order of thermal ion velocity. For such a rapid motion, the plasma in the magnetic field can be fully described by the model of an ideally conducting liquid. Therefore, rapidly developing large-scale instabilities are known as the magnetohydrodynamic instabilities. They develop owing to the diamagnetic properties of plasma. The plasma tends to shift towards a decreasing field; therefore, if the plasma is in a region where the field decreases in the outward direction from the plasma boundary, this boundary can prove to be unstable. As discussed above, in open-ended machines, we can generate the minimum-B magnetic field which provides for the magnetohydrodynamic stability of the plasma in the machine.

For the closed traps, however, it has been established that it is impossible to generate a magnetic field increasing outwards from the plasma boundary at each surface point of a toroidal plasma configuration. The grad $|B|$ component normal to the plasma surface has different signs at different points of the boundary. For instance, in Tokamaks the value of B decreases outwards at the outside of the plasma ring and increases at the inside of the ring. Thus, a question arises whether the plasma can flow towards a decreasing field in the form of individual "tongues". To answer this

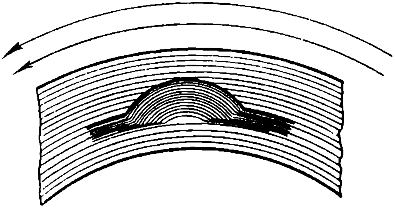

Fig. 22.
"Tongue" on the plasma surface.

question, we have to differentiate between a high-pressure plasma for which $p \sim B^2/8\pi$ and a low-pressure plasma for which $p \ll B^2/8\pi$. On the surface of the high-pressure plasma, there can develop local perturbations in the form of "tongues" (Fig. 22). Since the field is "frozen" into the plasma, the development of the tongue results in an increase of magnetic energy and in curving of field lines. An appropriate work will be done by the expanding high-pressure plasma at the expense of its thermal energy. If the tongue extends towards a decreasing field, it will extend further so that the plasma boundary will be unstable. In this case instability has a local nature since it is determined by the local field geometry. In toroidal traps there always exist parts of the plasma ring where the field geometry encourages the development of tongues. Hence, the high-pressure plasma is unstable in such systems.

If $p \ll B^2/8\pi$ as is the case in modern toroidal machines (for instance, Tokamaks and Stellarators),

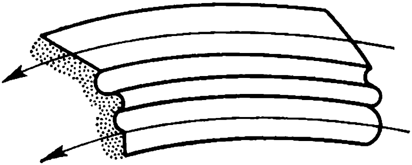

Fig. 23.
Development of flutes on the plasma surface.

plasma perturbations cannot distort noticeably the shape of the field lines. The plasma just lacks an extra thermal energy to do this. Hence, small local perturbations of the tongue-type are stabilized automatically and the only perturbations inside the plasma or at its boundary that can occur are the rearrangements of the whole field tubes and the formation of flutes (Fig. 23).

The plasma which filled the field tube formed by a thin bunch of field lines tends to expand and will, therefore, migrate towards the side where the tube volume will increase. This volume is $\int \delta S dl$, where δS is the area of the tube cross section and dl is the elementary length of the field line. Since the magnetic flux $\delta\Phi$ is constant along the length of the tube, we have

$$\delta v = \int \delta S B dl/B = \int \delta\Phi dl/B = \delta\Phi \int dl/B \quad (72)$$

The flux $\delta\Phi$ remains constant for all movements of the tube. Hence, the tube volume varies as $\int dl/B$.

Since a plasma, as any other gas, tends to increase its volume in motion of a tube with the plasma, the parameter $u = -\int dl/B$ plays a part similar to that of the potential energy. Movements of individual plasma elements with the exchange of the field tubes are known as rearrangement or convective deformations. When they reach the plasma boundary with an external field, the plasma surface acquires a fluted structure oriented along the field lines. These are the "flute" instabilities briefly mentioned above.

Making use of the concept of "potential energy" u (this is a purely arbitrary term), we can formulate for the low-pressure plasma the stability criterion with respect to flute instabilities. The plasma boundary is stable if u increases along the field tube with the plasma towards its intersection with the plasma boundary, that is, if the following inequality is satisfied:

$$\delta \int dl/B < 0 \tag{73}$$

The variation of the integral is taken along the perpendicular to the plasma surface between two infinitesimally close field lines. If the inequality is not satisfied, the boundary is unstable.

The stability condition (73) means physically that the necessary and sufficient condition of stability is the increase of the averaged field along the field line outwards from the plasma boundary. This is a modified minimum-B concept. Note that

according to Eq. (72) $\int dl/B = dv/\delta\Phi$. Therefore, the quantity $\int dl/B$ can also be named the specific volume of the field tube.

To be able to use the above stability criterion, we have, at first, to eliminate the uncertainty in the expression for u, where no limits of integration along the field line are given. There is no uncertainty when the field lines are closed at the plasma surface. In this special case, the integral $\int dl/B$, clearly, must be taken along the length of the field line. We can introduce here the following improved definition of the potential energy for the magnetic surface with closed field lines:

$$u = -1/N \oint dl/B \tag{74}$$

Here N is the number of turns around the magnetic system after which the field line is closed upon itself. This can be naturally extended to the case of closed field lines:

$$u = \lim 1/N \int dl/B \tag{75}$$

Here N and the length of integration tend to infinity. The parameter u can be shown to be a definite characteristic of the magnetic surface. Therefore, the expression for u can be rewritten to include only the parameters of the given magnetic surface so that any uncertainty in the length of integration is eliminated.

The volume V corresponding to a closed magnetic surface can be treated as a function of the flux Φ_θ of the longitudinal magnetic field passing through its cross section: $V = V(\Phi_\theta)$. Using this notation we can write the potential energy as $u = -V'(\Phi_\theta)$, so that the stability condition can be written in the following form:

$$V''(\Phi_\theta) < 0 \tag{76}$$

The form (76) of the stability condition is valid not only for the systems in which the field lines are closed, but also for the general case when the field lines on the magnetic surface are infinite.

For each plasma configuration we can plot a "magnetic well", that is, the curve $u(\rho)$, where ρ is the distance between a given surface and the magnetic axis (at a small distance from the axis we have $\Phi_\theta \sim \rho^2$). The relative depth of the magnetic well is

$$\Delta u / | u_0 | = [u(\rho_0) - u(0)] / | u(0) |$$

where $u(\rho_0)$ and $u(0)$ are the values of u at the plasma surface and at the magnetic axis. The higher the parameter $\Delta u / | u_0 |$ the better the stability of plasma with respect to the convective instabilities. If this parameter is negative, that is, the system has a magnetic "peak" instead of the well, the plasma is unstable. Analysis of the function $u(\rho)$ (or the equivalent function $V'(\Phi_\theta)$) shows that the existence or lack of the magnetic well is determined by the arrangement of magnetic surfaces with respect to the magnetic axis.

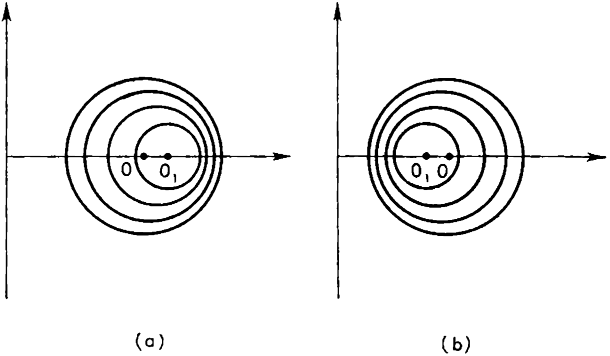

Fig. 24.

Displacement of magnetic surfaces inwards (*a*) and outwards (*b*) from the magnetic axis.

Fig. 24*a* and *b* illustrates two different arrangements of magnetic surfaces in toroidal machines: at the left diagram the cross sections of magnetic surfaces are shifted inwards from the magnetic axis, that is, towards the geometrical centre of the system as a whole and the displacement is the larger the larger is ρ; at the second diagram the situation is reversed and the magnetic surfaces are displaced outwards from the magnetic axis. In the first case u increases with ρ; hence, there is a magnetic well and the plasma is stable. In the second case there is no magnetic well and the plasma is unstable. These results follow from the rigorous theoretical analysis but qualitatively they are

almost self-evident. If the magnetic surface is displaced inwards, it gets into a stronger longitudinal field. Therefore, the flux increases faster than the volume and $V''(\Phi_\theta)$ must be negative, which means that the plasma is stable. When the magnetic surfaces are displaced outwards, the effect must be reversed.

Let us illustrate the above discussion by the examples of Tokamaks and Stellarators. As mentioned above, in Tokamaks the magnetic surfaces are displaced inwards with respect to the magnetic axis. Therefore, the plasma ring in Tokamaks must be stable with respect to convective magnetohydrodynamic instabilities and, in particular, to flute instabilities at the surface. However, in the existing Tokamaks the relative depth of the magnetic well is very small (of the order of a few per cent).

Analysis of the geometry of magnetic surfaces in stellarators shows that magnetic surfaces in them are displaced outwards. Hence, we have $V''(\Phi_\theta) > 0$ and the plasma ring can be unstable.

12.

SHEAR STABILIZATION

The magnetohydrodynamic instabilities can be suppressed not only by means of a magnetic well but also by the so-called shear, that is, crossing of the field lines. If there is the shear in a system where the field line rotates around the magnetic axis, this means that the rotation angle of the field

lines is a function of r, that is, it varies from the magnetic axis to the periphery of the plasma ring. The rotation angle depends on the geometry of the magnetic system in the presence of a plasma ring. For instance, in Tokamaks it can readily be seen that for $a/R \ll 1$ the angle of rotation of the field line around the magnetic axis is (per unit length)

$$\mu = B_\varphi / r B_\theta$$

The shear exists when μ is a function of r, that is, if $d\mu/dr$ is non-zero. To compare shears in different plasma systems we have to make use of the dimensionless parameters. In the publications on the stability theory the usual shear parameter is

$$\theta_s = r^2 d\mu/dr \tag{77}$$

In the existing experimental machines the shear θ_s is not more than a few per cent.

It is widely believed that the shear is some universal technique for suppressing a very extensive range of plasma instabilities. Let us attempt illustrating the shear stabilization by discussing the magnetohydrodynamic instabilities.

Assume that we have a plasma with a low pressure $8\pi p/B^2 \ll 1$. If the shear θ_s is close to zero and there is no magnetic well, then the convective instabilities can freely travel along the radius by rearrangement of the field tubes with the plasma and come out on the surface of the plasma ring in the form of flute instabilities. With an increasing shear such instabilities will be stabilized, since

a radial displacement of plasma in this case will result in a strong distortion of the field. When $\theta_s \neq 0$, then a deformation which had at a certain surface the form of a protrusion coinciding with the field line must curve at the neighbouring magnetic surface since the field lines have rotated. The result must be a plasma tongue of a complicated shape. Its edges are as though fixed along the internal field line and the radially farthest ridge is parallel to the external field line. The extra magnetic energy required for this distortion of the field's shape can be obtained only at the expense of the work of plasma pressure (remember that the field lines are frozen into the plasma). Hence, with a finite shear, perturbations in the plasma can be stabilized if the ratio $8\pi p/B^2$ is small enough. It can happen, however, that with a low shear a hot plasma can be stabilized only at a very low density.

13.

OTHER PLASMA INSTABILITIES

The above discussion dealt with the rapidly developing large-scale instabilities for which plasma is an ideal conductor. The real plasma has a finite conductivity. Therefore, for slowly developing instabilities, the field lines cannot be assumed to be frozen into the plasma and are not carried with it. The "thawing out" of the field lines must

facilitate propagation of perturbations in the plasma and can make the effectiveness of the above stabilization mechanisms insufficient. This extends the range of possible plasma instabilities. This can be illustrated by the so-called current-convective, or "corkscrew" instability which develops in the plasma when there is a current parallel to the magnetic field. This type of instability has the following origin. A temperature fluctuation in a plasma with a longitudinal current changes the electric field in the plasma owing to the temperature dependence of conductivity. Under certain conditions, this can give rise to a drift motion perpendicular to **B** which involves the entire plasma layers. If the plasma has a temperature gradient, the drifts can, in their turn, amplify the initial thermal fluctuations owing to the replacement of colder layers by hotter ones.

The instabilities due to finite conductivity of the plasma are usually known as dissipative instabilities. The perturbations with these instabilities are large in scale but have relatively low rise increments. The dissipative instabilities can decrease the efficiency of plasma heating owing to increasing heat losses. But with increasing plasma temperature, their significance decreases (since the plasma conductivity increases with temperature).

The effect of dissipative instabilities on energy retention in a hot plasma seems to be practically confined to the electron temperature range below 10^6 degrees. When we have passed the initial heating stage and generated a plasma with a higher

electron temperature, the dissipation processes practically cease to affect the plasma stability.

If we have managed to produce a plasma configuration in which there is no development of magnetohydrodynamic instabilities including the dissipative ones, it still does not follow that diffusion and heat conductivity in this plasma are due only to pair collisions of charged particles. There is another type of instabilities which can make the confined plasma "chronically sick" and result in its premature "demise" following spreading out or cooling. These are the so-called drift instabilities.

Before discussing the mechanism of drift instabilities, I would like to mention a problem of a more general nature. In a plasma there can develop and propagate a variety of oscillations and waves in a very wide frequency range. For instance, when electrons are displaced with respect to ions, this gives rise to high-frequency electron oscillations with an angular frequency $\omega_0 = \sqrt{4\pi ne^2/m_e}$. These oscillations can propagate in the plasma as longitudinal electrostatic waves. There can also be generated waves of a much lower frequency due to the longitudinal oscillations of ions—the so-called ion sound. Along the field lines of the magnetic field, in the plasma there can propagate transverse electromagnetic waves (the so-called Alfven waves) whose velocity is $B/\sqrt{4\pi nm_i}$ and is much lower than the light velocity when the plasma density is high.

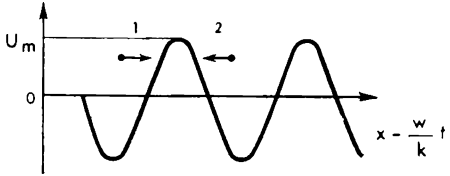

Fig. 25.
Interaction between a particle and a wave.

We shall not go here into the details of wave problems but mention only one general question, namely, the interaction between waves and particle flows in the plasma. The character of this interaction will determine amplification or attenuation of waves of a given type. Assume that a longitudinal wave parallel to the axis x has been generated in the plasma. The wave process here is the periodic oscillations of the electric potential travelling with the velocity $v_f = \omega/k$, where ω is the cyclic frequency and k is the wave vector ($k = 2\pi/\lambda$).

Let us analyze the interaction between the plasma particles and this running periodic potential barrier. Electrons or ions whose velocity component along the axis x greatly differs from v_f will have small oscillations of velocity in the region of variable potential with the conservation of the mean energy. But if a particle has the velocity component v_x close enough to v_f, the particle and the wave will exchange energy. To understand this energy exchange, let us make use of the reference system which travels with the wave. Fig. 25 shows variation of the potential

energy of the particle in this system. Let the amplitude of oscillations of the potential energy be u_m. The component of the kinetic energy of the particle along the axis x in the travelling reference system is $m(v_x - \omega/k)^2/2$. If this energy is less than u_m, the particle will be reflected by the potential barrier. We have to distinguish between two cases here. In the first case we have

$$\omega/k+\sqrt{2u_m/m} > v_x > \omega/k \tag{78}$$

and in the second case we have

$$\omega/k > v_x > \omega/k-\sqrt{2u_m/m} \tag{79}$$

In the first case the particle *1* catches up with the wave and in the second case the wave catches up with the particle *2*. Following the collision with the potential barrier, the velocity component v_x of the first particle will reverse its sign and will be $\omega/k - v_x$. In the laboratory reference system it will be $2\omega/k - v_x$. Eq. (78) shows that this velocity is less than v_x. Thus, the particle's velocity has decreased following the collision, so that a part of its kinetic energy has been transferred to the wave. On the contrary, the collision of the second particle with the potential barrier will increase its velocity owing to the energy exchange with the wave. This simple analysis shows that the amplitude of the longitudinal electric wave in the plasma will increase or decrease depending on the majority of the plasma particles catching up with the wave or falling behind it. In other words, amplification or attenuation of the wave will be determi-

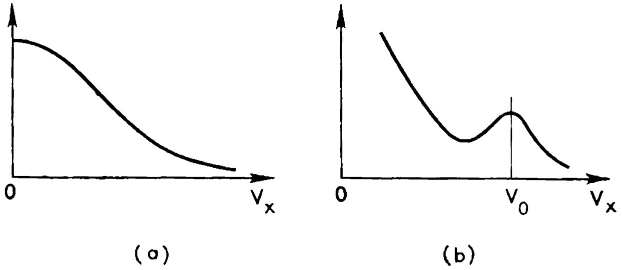

Fig. 26.

Velocity distribution of particles:
a — Maxwell distribution;
b — distribution in the presence of a beam.

ned by the shape of the velocity distribution of particles for the velocity component parallel to the wave propagation direction.

Fig. 26 presents two different velocity distributions for a given direction (the first distribution corresponds to the Maxwell velocity distribution). Here we have $df/dv_x < 0$ in the whole velocity range. Hence, for any phase velocity of the wave and any amplitude of potential oscillations, there will be more particles that fall behind than those that catch up with the wave. Therefore, the wave will be attenuated. This mechanism is known as the Landau damping. The second velocity distribution function has the maximum at $v_x = v_0$. To the left of this maximum, we have $df/dv_x > 0$. Hence, the waves with the phase velocities in this range

will be amplified owing to the energy transfer from the particles.

Note that if the velocity distribution for the velocity component v_x has a maximum at $v_x \neq 0$, this can be interpreted as a result of a directed flow of particles in the plasma. (This means that the distribution in Fig. 26*b* can be represented as a superposition of a steadily decreasing function and an individual peak.)

It is clear that the amplification of waves in the plasma is due just to the energy transfer from the particles of the directed flow. The energy transfer from a directed flow of electrons or ions to waves in the plasma decelerates the flow and increases the periodic electric field in the plasma. Various specific effects due to this physical mechanism are known as the beam instabilities. They are encountered in various phenomena occurring in plasma. However, the development of such an instability does not mean by itself that the thermal insulation of plasma has been disrupted and the diffusion rate has increased. The drifts contributing to the loss of particles and energy from the plasma can develop only if the electric field of the wave due to the beam instability has a component perpendicular to **B**.

This is just the case for the beam instabilities discussed below. These instabilities are, in a sense, universal, since they are due to the fact that the plasma has non-uniform density and temperature in the direction perpendicular to the magnetic field. Any confined plasma configuration is not uniform in this respect for the simple reason that near the

plasma boundary the concentration should sharply decrease and the temperature should also drop.

Let us consider the simplest plasma configuration—a long cylindrical column of a low-pressure plasma in the longitudinal magnetic field. We shall assume the magnetic field to be uniform. The equilibrium equation yields

$$j_{\perp} = c/B \mid \text{grad}\, p \mid \tag{80}$$

where $j_{\perp}$ is the density of diamagnetic current in the plasma. In the case of cylindrical symmetry, Eq. (80) can be written as

$$j_{\perp} = c/B \left| \frac{dp}{dr} \right| \tag{81}$$

Here the current will be perpendicular to the pressure gradient and the vector **B**, that is, it will flow in the azimuthal direction. Since the diamagnetic current is due to the Larmor motion of electrons and ions, we can write

$$j_{\perp} = n_e (\mathbf{u}_i - \mathbf{u}_e)$$

where $\mathbf{u}_i$ and $\mathbf{u}_e$ are the averaged drift velocities of electrons and ions. The drift velocities have also the azimuthal direction. We have here

$$u_i = \frac{c}{n_e B} \left| \frac{dp_i}{dr} \right|; \quad u_e = \frac{c}{n_e B} \left| \frac{dp_e}{dr} \right| \tag{82}$$

If electrons and ions have close temperatures, so that we can introduce the general plasma temperature T, we can write

$$u_i \approx u_e \approx \frac{ckT}{eB} \left| \frac{1}{p} \frac{dp}{dr} \right| = \frac{ckT}{eaB} \tag{83}$$

Here a is the characteristic linear size of pressure non-uniformity in the plasma:

$$a^{-1} = -\frac{1}{p}\frac{dp}{dr}$$

If the concentration drops from the axis to the boundary of the plasma column more sharply than does the temperature, we have

$$a^{-1} \approx -\frac{1}{n}\frac{dn}{dr}$$

If the temperature drop is more sharp, we obtain

$$a^{-1} \approx -\frac{1}{T}\frac{dT}{dr}$$

Analysis results suggest that when in the plasma there develop periodic perturbations of the wave character which propagate at a small angle to the azimuth, the phase velocity of this wave ω/k_φ is ckT/eaB and such waves are known as the drift waves. Since the drift azimuthal velocities of particles are close to the phase velocity of waves propagating in the same direction, the particles should be expected to interact strongly with each other so that conditions can be established for rapid amplification of oscillations. Increase in the amplitude of oscillations results in development of strong azimuthal electric fields, therefore, when the plasma has no uniform density and temperature, there must develop radial flows of heat and particles which shorten the lifetime of a confined plasma configuration. That is the effect of drift instabilities.

Detailed theoretical analysis shows that the amplification of drift waves due to non-uniform plasma density is effectively inhibited by a high enough shear. The drift instability due to the temperature gradient is highly dangerous. Instabilities of this type are known as drift temperature instabilities. Such instabilities develop when $d(\ln T)/d(\ln n) > 1$ and considerably increase the heat losses from the plasma. Though an additional heat loss due to the drift temperature instability in closed toroidal systems can be sharply reduced by shear, it still can be higher than the heat losses due to the classical mechanism of pair collisions. However, though the available theory can predict the conditions for development of the drift temperature instability, the anomalous heat conductivity coefficient can still be calculated only to an order of magnitude accuracy. Very roughly, the anomalous heat conductivity coefficient in Tokamaks can be estimated to be 10-20 times as high as the coefficients predicted by the classical theory taking into account the role played by the confined particles. The experimentally measured plasma heat losses lie somewhere between the classical results and the calculated anomalous heat losses. Therefore, the drift instability theory cannot be said to be well substantiated by experiments. This inconsistency of the theory and experiment reflects the current state of research into plasma instabilities. The theoretical classification of various instability types in the plasma has been in the main completed. The mechanisms of instabilities have been analyzed and the condi-

tions have been found under which small perturbations for a given instability decrease or increase. However, the linear-approximation analysis of such processes is only the first stage of the theory.

The next, much more difficult stage is to determine the coefficients of diffusion and heat conductivity in plasma with developing instability. Such a plasma is typically referred to as the turbulent plasma. There has been made only a preliminary analysis of diffusion and heat conductivity in the turbulent plasma yielding primarily qualitative results which are difficult to compare with experimental results. The only processes that are comparatively simple to analyze are transport phenomena under the effect of magnetohydrodynamic instabilities. In this case large regions of plasma must move with the velocities of the order of the thermal velocity of ions. If the magnetohydrodynamic instabilities are eliminated, all the processes in the plasma are smoothened down. Only drift-type instabilities remain, they give rise to plasma turbulence and increase the rates of the transport processes. To make, at least, rough estimates of the rates of diffusion and heat transfer daring assumptions have to be made, for instance, the hypothesis of the "relay" transfer of perturbations in the plasma from one magnetic surface with closed field lines to another magnetic surface of the same type. However, at the very best, such an approach yields only qualitative results.

On the other hand, we should note that we still cannot make accurate enough measurements of the

diffusion and heat conductivity coefficients in experiments. To find these parameters we must know the distributions of density and temperature over the cross section of the plasma and to compare the measurements with the predicted values, we have to know in detail the geometry of the magnetic field in the plasma. The available experimental techniques are largely unsuitable for that. Therefore, we have to be content with determining such general characteristics as the mean lifetimes of particles in the plasma and the mean time of energy retention in the plasma. These characteristics must be compared with the predicted values proceeding from fairly arbitrary assumptions about distribution of density and temperature in the plasma. So the reader can see that the physics of plasma is still a rather crude science and it is only the variety of sophisticated equations derived by theoreticians that to a certain extent can mask its immaturity.

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